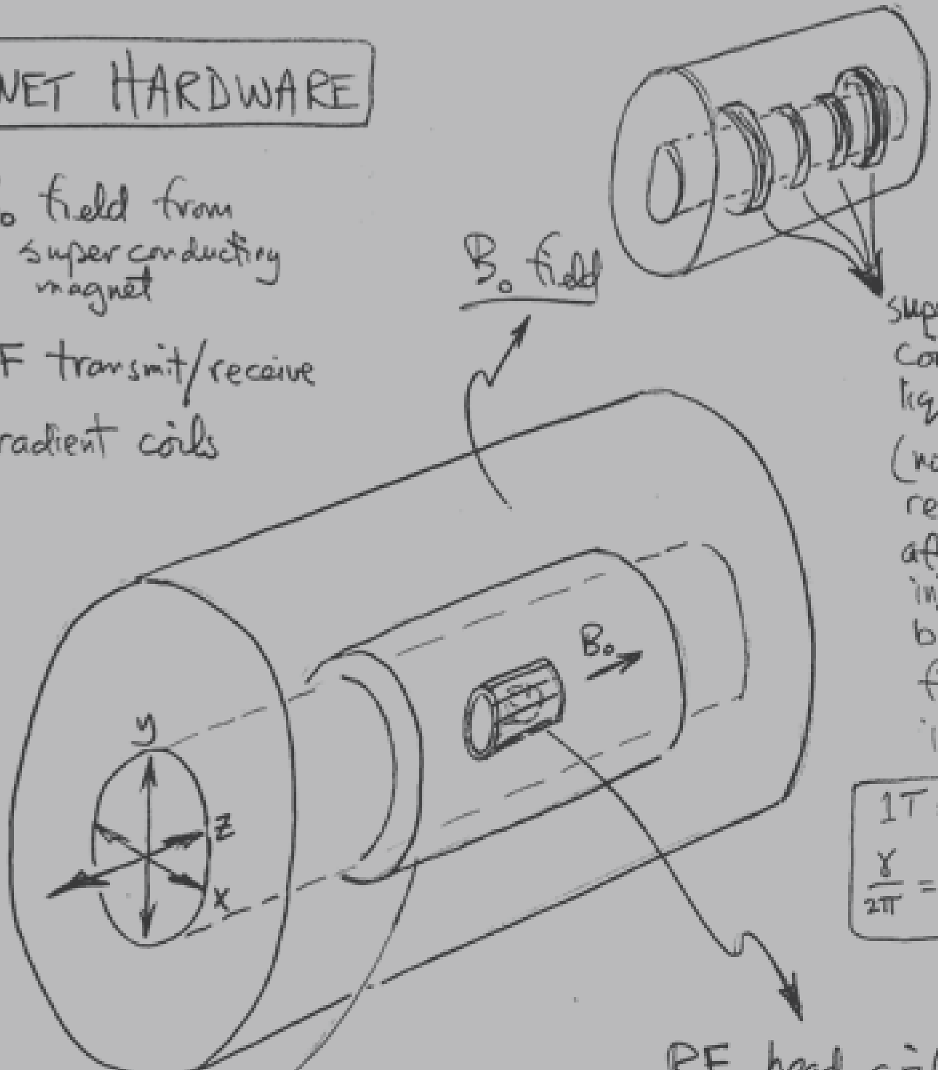
[redacted]

Φ

MAGNET HARDWARE

- B_0 field from superconducting magnet
- RF transmit/receive
- gradient coils

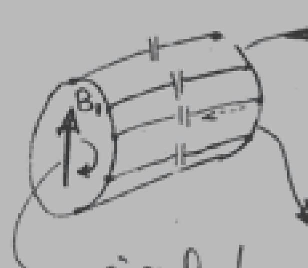


superconducting coils in liquid helium (no power required after current injected to bring up field using induction)

$1T = 10{,}000$ Gauss

$\frac{\gamma}{2\pi} = 42.6$ MHz/T

RF head coil



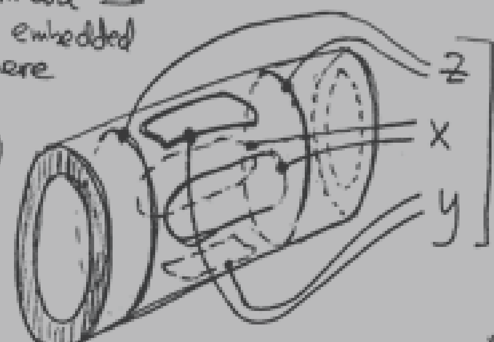
RF transmitter (10 KW)

RF receiver

circularly polarized B1 field rotating $\perp$ to B0 at Larmor freq. (B1 is several orders of magnitude smaller than B0)

body gradient coils

shim coil also embedded in here (not shown)



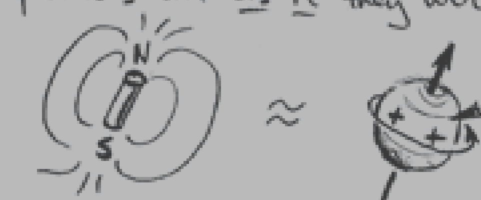
three 0.5 million watt amplifiers to add ramps to B_0^z field

↳ usu. water cooled

SPIN & PRECESSION

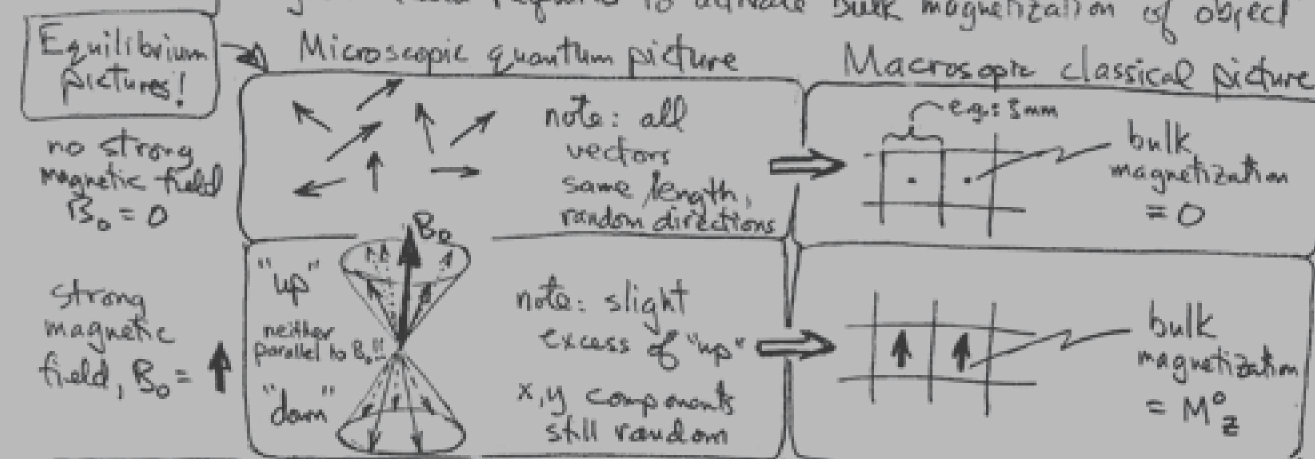
N.B.: compared to top + gravity magnetic relaxation is:
- frictionless spin
- signed gravity (can change direction of precession)
- neighbor bumping causes decay

- nuclei act like spinning charges
  - ↳ nuclei w/ odd atomic weight or odd proton numbers (e.g., hydrogen)
- moving charges create a magnetic field
- protons act as if they were spinning (tho classically, this would cause energy to be lost, resulting in spin-down)



classical picture: charges on surface of spinning sphere make current loops, creating magnetic dipole field

- strong magnetic field required to activate bulk magnetization of object



Precession
- add to equilibrium pictures above!! (distinguish precession from spin)
- treat classically, like top

$$\underbrace{2\pi f_0}_{\text{Larmor freq (63 MHz)}} = \underbrace{\omega_0}_{\text{same in radians/sec}} = \underbrace{\gamma}_{\text{gyro-magnetic ratio}} \underbrace{B_0}_{\text{static field (1.5T)}}$$

like top: precession faster w/ more gravity

- bulk magnetization: parallel to B_0

$$M_z^0 = |\vec{M}| = \frac{\gamma^2 h^2 B_0 N_s}{4 K T_s}$$

two non-constants

- γ – gyromagnetic ratio (const.)
- h – Planck's constant
- B_0 – M_z^0 proportional to mag strength
- N_s – " " to number spins
- K – Boltzmann constant
- T_s – abs. temp sample

Bloch - 1

BLOCH EQUATION

- time-dependent behavior of $\vec{M}$ (vector sum of all microscopic magnetization vectors) in the presence of an applied magnetic field (excitation & relaxation)

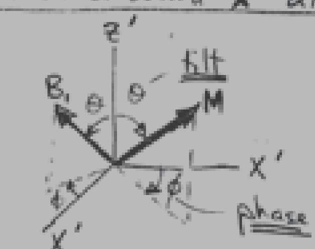
(laboratory frame)
$$\underbrace{\frac{d\vec{M}}{dt}}_{\text{change in mag vector}} = \overbrace{\underbrace{\vec{M}}_{\substack{\text{existing}\\ \text{mag vector}\\ \text{sum}}} \times \underbrace{\gamma\vec{B}}_{\substack{\text{applied field}\\ \text{(typically rotating)}}}}^{\substack{\text{precession: } B = B_0\\ \text{excitation: } B = B_0 + B_1}} - \underbrace{\overbrace{\frac{M_x\vec{i} + M_y\vec{j}}{T_2}}^{\substack{\text{"transverse"}\\ \text{"spin-spin"}}} - \overbrace{\frac{(M_z - M_z^o)\vec{k}}{T_1}}^{\substack{\text{"longitudinal"}\\ \text{"spin-lattice"}}}}$$

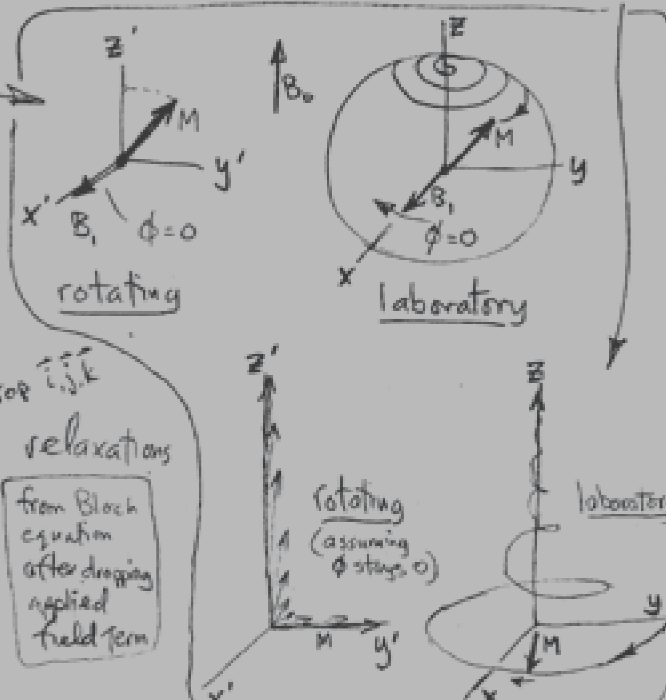
M_z^o — equilibrium $\vec{M}$ value in only the B_0 field; $\vec{k}$ — unit vector in z-direction

↳ these can be ignored during short excitation, so magnetic vector stays same length as it spirals down (vs. relaxation, where it shrinks, then grows)

- in the Larmor-rotating coordinate system, a tilt w/o a phase shift for a standard B_1 excitation is rotation around **x**-axis

general case rotating (distinguish B_1 & M!)





- longitudinal and transverse relaxations (drop i, j, k)

(rotating frame)

M_L: $$\frac{dM_{z'}}{dt} = -\frac{M_{z'} - M_z^o}{T_1}$$

M_T: $$\frac{dM_{x'y'}}{dt} = -\frac{M_{x'y'}}{T_2}$$

from Bloch equation after dropping applied field term

- solution to equations above: time-dependent free precession eq's

lab frame: same!

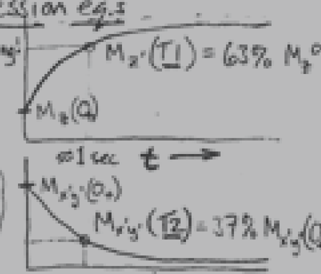
(rotating frame)

M_L: $$M_{z'}(t) = \underbrace{\overbrace{M_{z'}^o\left(1 - e^{-t/T_1}\right)}^{\text{re-growing from 0}}}_{M_{z'}^o\text{: initial mag}} + \overbrace{M_{z'}(0_+)e^{-t/T_1}}^{\text{leftover after pulse—decaying}}$$

M_T: $$M_{x'y'}(t) = \underbrace{M_{x'y'}(0_+)}e^{-t/T_2}$$

lab frame: times $e^{-i\omega_0 t}$

M_z, M_{xy} at time immediately after pulse

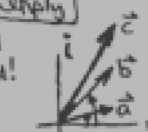


Bloch - 1a

BLOCH EQ. – MATRIX VERSION

complex multiply (3rd kind!)

$$\vec{c} = \vec{a}\cdot\vec{b} = \begin{bmatrix} b_x & -b_y \\ b_y & b_x \end{bmatrix}\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \left[a_x b_x - a_y b_y \quad a_x b_y + a_y b_x\right] \text{ (vector)}$$



- angles add
- magnitudes multiply

inner product (= dot product) (= scaled projection onto)

$$c = \vec{a}\cdot\vec{b} = \left[a_x \; a_y \; a_z\right]\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = a_x b_x + a_y b_y + a_z b_z \text{ (scalar)}$$

$$= \underbrace{|\mathbf{a}|}_{\text{length of } \mathbf{a}}|\mathbf{b}|\cos\theta$$

$p = |\mathbf{a}|\cos\theta$

$c = p\,|\mathbf{b}|$ → $c = p$ if $|\mathbf{b}| = 1$

outer product (= cross product)

$$\vec{c} = \vec{a}\times\vec{b} = \begin{bmatrix} 0 & b_z & -b_y \\ -b_z & 0 & b_x \\ b_y & -b_x & 0 \end{bmatrix}\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \left[a_y b_z - a_z b_y \quad a_z b_x - a_x b_z \quad a_x b_y - a_y b_x\right] \text{ (vector)}$$

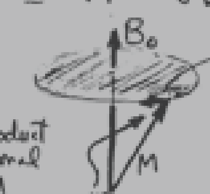
$$= |\mathbf{a}||\mathbf{b}|\sin\theta$$

Just Precession

$$\frac{d\vec{M}}{dt} = \vec{M}\times\gamma\vec{B}_0$$

(assume only z component of B_0 present)

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \gamma B_0 & 0 \\ -\gamma B_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$



cross product is orthogonal to B & M

precession

Solution: (M_x, M_y component coupling) $\omega_0 = \gamma B_0 \implies$ precession around z

$$\vec{M}(t) = \begin{bmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{bmatrix} = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t & 0 \\ -\sin\omega_0 t & \cos\omega_0 t & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} M_x^o \\ M_y^o \\ M_z^o \end{bmatrix} = \overset{\square}{R}_z(\omega_0 t)\,\vec{M}^o$$

Including Relaxation

$$\frac{d\vec{M}}{dt} = \begin{bmatrix} -1/T_2 & \gamma B_0 & 0 \\ -\gamma B_0 & -1/T_2 & 0 \\ 0 & 0 & -1/T_1 \end{bmatrix}\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_0/T_1 \end{bmatrix}$$

Solution:

$$\vec{M}(t) = \begin{bmatrix} e^{-t/T_2} & 0 & 0 \\ 0 & e^{-t/T_2} & 0 \\ 0 & 0 & e^{-t/T_1} \end{bmatrix}\overset{\square}{R}_z(\omega_0 t)\,\vec{M}^o + \begin{bmatrix} 0 \\ 0 \\ M_0\left(1 - e^{-t/T_1}\right) \end{bmatrix}$$

The human brain is, contrary to popular belief, not at all grey but pinkish. When preserved in formalin a dead brain will however turn grey, and this is likely why the brain so often has been portrayed as grey in popular culture.

Вопреки расхожему представлению, человеческий мозг вовсе не серый, а розовый. Находясь в формалине мёртвый мозг, тем не менее, становится серым, и, очевидно, именно поэтому мозг в популярной культуре часто имеет серый цвет.

Week-2

RF Field Polarization

B1, generated by RF coil

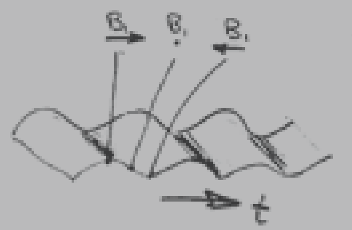
- polarization (i.e., direction) of magnetic (not electric!) field
- linearly polarized field

N.B.: $\vec{B}_1$ adds to much larger $\vec{B}_\emptyset$

B0, B1, B0+B1

B0+B1 wiggles

$$\vec{B}_1^{\,lin} = \underbrace{b_1^{\,lin}}_{\text{mag strength}} \cos \underbrace{\omega t}_{\text{phase angle}} \underbrace{\vec{x}}_{\text{unit vector}}$$



e.g., for $\vec{y}$: t

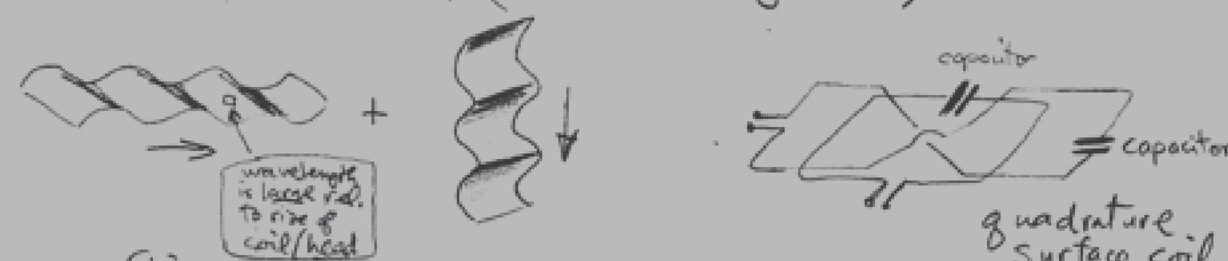
- circularly polarized field (quadrature)
  - made by adding two perpendicular lin. polarized
  - same freq/amp, 90° out of phase

circularly polarized rotates around B∅ direction:

B0+B1

$$\vec{B}_1^{\,cir} = B_1 \left(\vec{x} \cos \omega t - \vec{y} \sin \omega t \right)$$

left-circularly polarized field



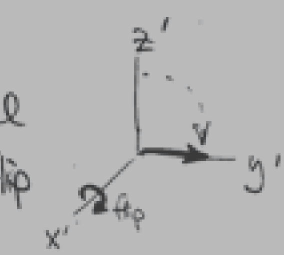
quadrature surface coil

- flipping around the x-axis versus flipping around y-axis is just a difference in phase of RF stim

⇓

across entire brain

typical 90° flip (around x-axis)



rotating frame

vs.

rotating frame

typical 180° flip (around y-axis)

180° flip around x-axis also gives spin echo but gives negative echo

SIGNAL EQUATION

$$\underbrace{\Phi(t)}_{\text{magnetic flux thru coil}} = \int_{obj} \underbrace{\vec{B}(\mathbf{r})}_{\text{mag. field detected (= generated) by coil geometry at each point in object}} \cdot \underbrace{\vec{M}(\mathbf{r},t)}_{\text{local magnetization of object (time-dependent)}} \, \underbrace{d\mathbf{r}}_{\text{position: } \mathbf{r} \to x,y,z}$$

for a particular instant in time vs. Bloch M&B which is change of **M** w/t

i.e., projection of mag. vector at each point onto coil magnetic field direction at each point, summed across object

$$V(t) = \underbrace{-\frac{\partial \Phi(t)}{\partial t}}_{\text{Faraday law of Induction}} = -\frac{\partial}{\partial t} \int_{obj} \vec{B}(\mathbf{r}) \cdot \vec{M}(\mathbf{r},t)\, d\mathbf{r}$$

- evaluate using free precession eqs. (solution to Bloch) ignoring relaxation
- rewrite w/ complex notation w/ time-dependence from lab frame Bloch

- ignore the change in the z-component of the magnetization since it changes so slowly compared to the free precession of x- and y-components: $\omega(\mathbf{r}) \ggg 1/T1(\mathbf{r})$
- this is why we can only record transverse magnetization, M_{xy}, but not longitudinal magnetization (M_L changes too slowly, so $V(t) = \frac{\partial \Phi}{\partial t} \approx 0$)

Laboratory frame Bloch solutions:
$M_L \to$ same
$M_T = M_{xy}(0_+) e^{-t/T_2} e^{-i\omega t}$

demodulate

$$\underbrace{S(t)}_{\text{complex: } \approx V(t)\text{'s}} = \int_{obj} \underbrace{B_{xy}(\mathbf{r})}_{\text{assume homogeneous (ignore)}} \underbrace{M_{xy}(\mathbf{r},0)}_{\text{lab frame transverse magnetization at time } t=0} \underbrace{e^{-i\,\delta\omega(\mathbf{r})t}}_{\omega_0 \text{ subtracted off by PSD}} \underbrace{d\mathbf{r}}_{\text{sum across object}}$$

δ = freq diff from γB_0

M_{xy} is now scalar, direction is here

spatially-dependent resonant freq in rotating frame — i.e. after subtraction of $\omega_0 = \gamma B_0$

ω_0 = freq = radians/sec
$\omega = \omega_0 + \delta\omega$ — gradients B1, ΔB0
$\omega_0 t$ = angle
$(\omega_0 + \delta\omega)t$ = angle
$\omega t + \phi$ = angle

$$S(t) = \int_{obj} M_{xy}(\mathbf{r},0)\, e^{-i\,\delta\omega(\mathbf{r})t}\, d\mathbf{r}$$

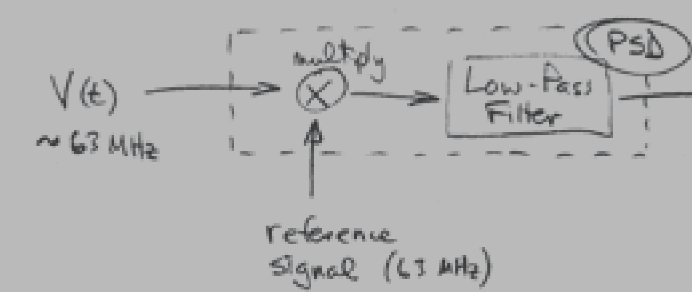
standard signal expression

i.e., at a single time point, RF signal is a sum across object of the phase angle of spins

phase angle in rotating frame
$\omega t = \frac{\text{radians}}{\text{sec}} \times \text{sec} = \text{radians}$ ($\phi = \int \omega\, dt$)
getting difference converts lab → rotating

PHASE-SENSITIVE DETECTION

how we get rotating frame



- method for moving very high frequency Larmor oscillations down to more tractable frequency range

demodulated signal $\propto$ reference $\cdot$ RF coil signal

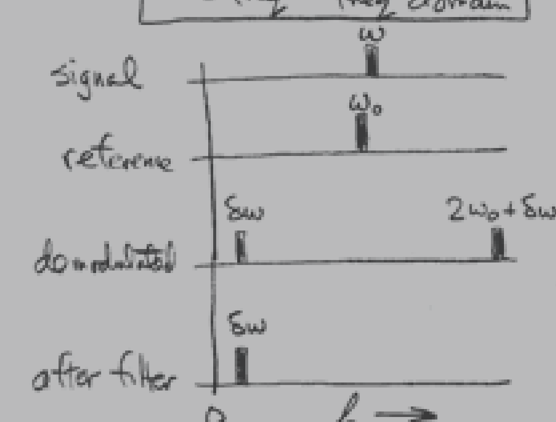
$\propto \sin(\omega_0 + \delta\omega)t \cdot \sin \omega_0 t$

$\sin a \sin b = \tfrac{1}{2}[\cos(a-b) - \cos(a+b)]$

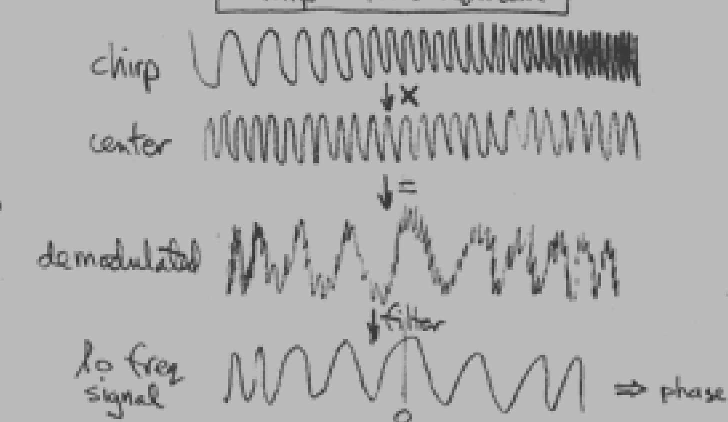
$$\propto \tfrac{1}{2}\big[\underbrace{\cos \delta\omega t}_{\text{diff between RF input \& ref}} - \underbrace{\cos(2\omega_0 + \delta\omega)t}_{\text{filter this one out w/ low pass filter}}\big]$$

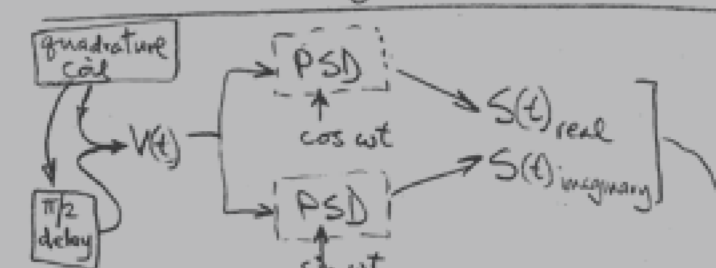
this signal is digitized

one freq – freq domain



chirp – time domain





$S(t)_{complex} = e^{i\,\delta\omega t}$

- two signals are made from a single receiving RF coil
- a quadrature coil can be treated the same way (OK to combine after adding π/2 phase, then PSD)
- quadrature coil has better S/N since noise in each part is uncorrelated (√2 better)

FID — free induction decay, $\underline{\underline{T_2^*}}$

T_2 — unrecoverable, rapid (intrinsic)
T_2^* — recoverable, static added

ignore space in object for now

- FID (free-induction decay) from an RF pulse w/ angle α

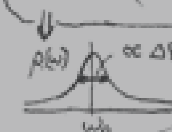
$$\underbrace{S(t)}_{\text{recorded signal (complex)}} = \underbrace{\sin\alpha}_{\text{amount of tip}} \int_{\omega=-\infty}^{\omega=\infty} \underbrace{\rho(\omega)}_{\text{spectral density function}} \underbrace{e^{-t/T_2(\omega)}}_{\text{freq dependent time-dependent intrinsic decay (spin-spin)}} \underbrace{e^{-i\omega t}}_{\sin,\cos} \underbrace{d\omega}_{\text{sum across all freq}}$$

- for a single freq:

$$S(t) = M_z^0 \sin\alpha \, e^{-t/T_2} \, e^{-i\omega_0 t} \quad \text{(one freq)}$$

Spectral density for "Lorentzian" mag. field inhomogen.

$$\rho(\omega) = M_z^0 \frac{(\gamma\Delta B_0)^2}{(\gamma\Delta B_0)^2 + (\omega-\omega_0)^2}$$



exponential fall off

- max FID amplitude at $t=0$: $S(0) = \overbrace{M_z^0}^{\text{bulk}} \overbrace{\sin\alpha}^{\text{flip angle}}$

- if field inhomogeneous (Lorentzian distribution)

for other dist., assume T_2^* is exp. approx. e.g., see bottom

$$\overbrace{S(t)}^{\text{complex}} = \pi \overbrace{M_z^0}^{\text{start } M_z} \overbrace{\gamma}^{\text{gyro}} \overbrace{\Delta B_0}^{\text{width inhomo.}} \overbrace{\sin\alpha}^{\text{tip angle}} \overbrace{e^{-t/T_2^*}}^{\text{decay}} \overbrace{e^{-i\omega_0 t}}^{\text{oscill.}}$$

(ΔB_0: since height $\rho(\omega)$ constant)

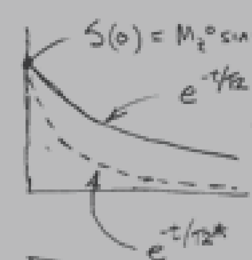
where $\underbrace{\frac{1}{T_2^*}}_{\substack{\text{decay rate}\\ \text{transverse w/}\\ \text{inhomogeneous}\\ B_0 \text{ field}}} = \underbrace{\frac{1}{T_2}}_{\substack{\text{spin}\\ \text{interaction}\\ \text{rate}}} + \frac{1}{T_2'} \rightarrow \gamma\Delta B_0$

decay rate $1/\tau$ is increased

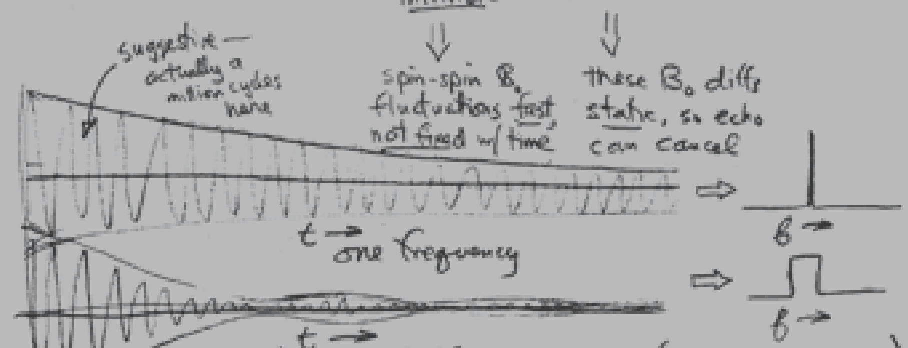
$\frac{1}{T_2}$ ⇓ unrecoverable "intrinsic" ⇓ spin-spin B_0 fluctuations fast, not fixed w/ time

$\frac{1}{T_2'}$ ⇓ recoverable ⇓ these B_0 diffs static, so echo can cancel

* basic FID envelope is proportional to e^{-t/T_2^*}



both of these would be approximated by T_2^*

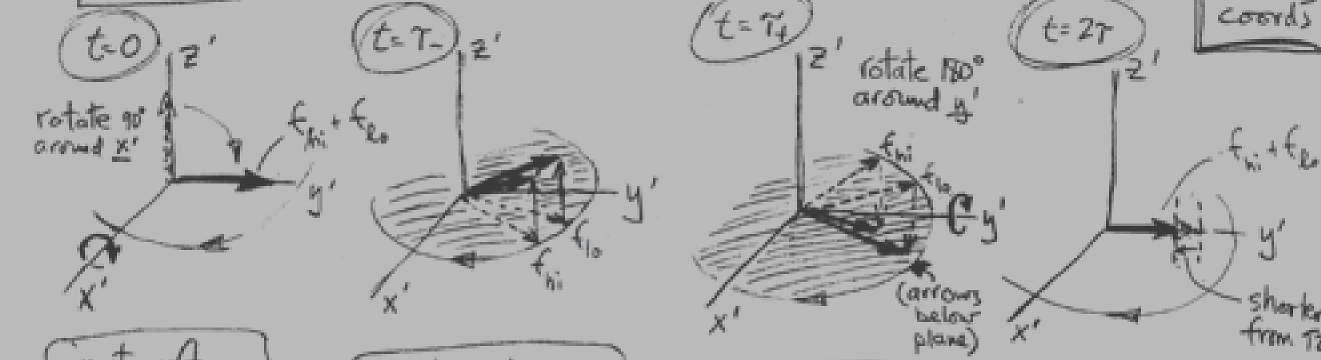


typical time course 10's of msec vs. each precession cycle, which is 10's of nsec

ECHOES — spin echo

$90° - \tau - 180°$, T_2/T_2^* & echo

rotating coords



just after $90°x'$ pulse $f_{lo} + f_{hi}$ have same phase

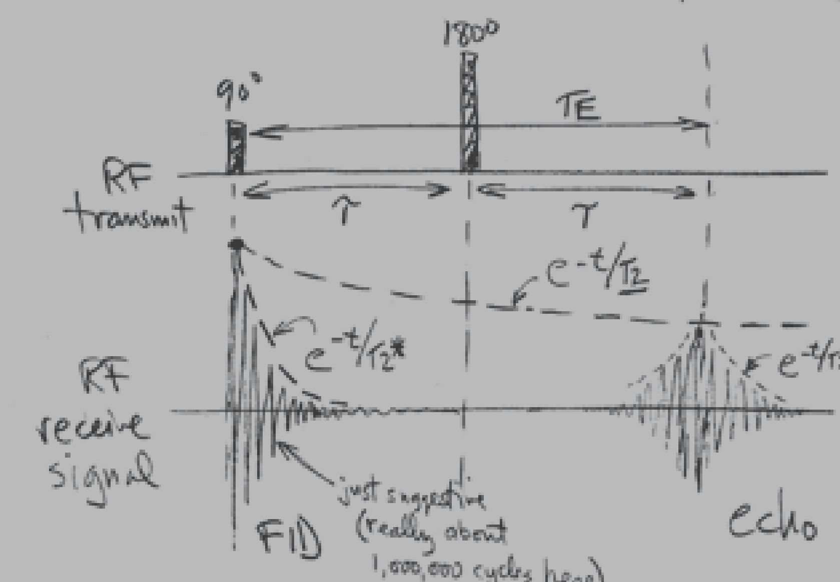
relaxation + phase dispersion of $f_{lo} + f_{hi}$ (both from $B > B_0$)

just after $180°y'$ pulse

(y' pulse like x' pulse but RF has $+90°$ phase)

echo caused by re-phasing of $f_{lo} + f_{hi}$ (w/ decay due to T_2)

- remember [RF just tips vector(s) while retaining length; relaxation includes tips *and* shrinks (and grows for echo)
- $180°\ \underline{\underline{x'}}$ pulse works, too, but echo will be $+\pi$ phase (left side in figs above)
- echos generated even if second pulse not 180° (see next)



- FID decay (and echo growth/decay) described by T_2^*, from inhomogeneity
- reduction in height of echo compared to initial described by T_2, echo fixes the "star"

ECHOES – spin echo – $\alpha_1 - \tau - \alpha_2$

(both pulses along y' for simplicity)

general transforms

effect of α-pulse

$$M_{x'} \rightarrow M_{x'} \cos\alpha - M_{z'} \sin\alpha$$
$$M_{y'} \rightarrow M_{y'}$$
$$M_{z'} \rightarrow M_{x'} \sin\alpha - M_{z'} \cos\alpha$$

effect of τ delay

$$M_{x'} \rightarrow (M_{x'} \cos\omega\tau + M_{y'} \sin\omega\tau)\, e^{-\tau/T_2}$$
$$M_{y'} \rightarrow (-M_{x'} \sin\omega\tau + M_{y'} \cos\omega\tau)\, e^{-\tau/T_2}$$
$$M_{z'} \rightarrow M_z^0 (1 - e^{-\tau/T_1}) + M_{z'} e^{-\tau/T_1}$$

---

immediately after α_1 pulse

$$M_{x'}(\omega, 0_+) = -M_z^0(\omega) \sin\alpha_1$$
$$M_{y'}(\omega, 0_+) = 0$$
$$M_{z'}(\omega, 0_+) = M_z^0(\omega) \cos\alpha_1$$

for one isochromat of freq. ω

after τ delay

$$M_{x'}(\omega,\tau) = -M_z^0(\omega) \sin\alpha_1 \cos\omega\tau\, e^{-\tau/T_2}$$
$$M_{y'}(\omega,\tau) = M_z^0(\omega) \sin\alpha_1 \sin\omega\tau\, e^{-\tau/T_2}$$
$$M_{z'}(\omega,\tau) = M_z^0(\omega) \left[1 - (1 - \cos\alpha_1) e^{-\tau/T_1}\right]$$

immediately after α_2 pulse (no effect on $M_{y'}$; rewrite y; combine x and y eq.s)

$$M_{x'y'}(\omega, \tau_+) = M_z^0(\omega) \sin\alpha_1 \left(\sin^2\frac{\alpha_2}{2} e^{-i\omega\tau} - \cos^2\frac{\alpha_2}{2} e^{i\omega\tau}\right) e^{-\tau/T_2} - M_z(\omega)\left[1 - (1-\cos\alpha_1) e^{-\tau/T_1}\right] \sin\alpha_2$$

*** time dependent free precession around z'** (rewrite $M_{x'y'}(\omega, \tau_+)$)

$$M_{x'y'}(\omega, t) = M_{x'y'}(\omega, \tau_+)\, e^{-(t-\tau)/T_2} e^{-i\omega(t-\tau)}$$

terms

$$= M_z^0(\omega) \sin\alpha_1 \sin^2\frac{\alpha_2}{2} e^{-t/T_2} e^{-i\omega(t-2\tau)} \quad (1)$$
$$- M_z^0(\omega) \sin\alpha_1 \cos^2\frac{\alpha_2}{2} e^{-t/T_2} e^{-i\omega t} \quad (2)$$
$$- M_z^0(\omega)\left[1 - (1-\cos\alpha_1) e^{-\tau/T_1}\right] \sin\alpha_2\, e^{-(t-\tau)/T_2} e^{-i\omega(t-\tau)} \quad (3)$$

– for a large num of freq's:

[terms (2) & (3) are dephasing → FID of echo
term (1) rephasing → rephase at $t = 2\tau$]

echo signal from (1)

$$S(t) = \sin\alpha_1 \sin^2\frac{\alpha_2}{2} \int_{-\infty}^{\infty} \rho(\omega) e^{-t/T_2(\omega)} e^{-i\omega(t - T_E)}\, d\omega$$

peak ampl

$$A_E = \sin\alpha_1 \sin^2\frac{\alpha_2}{2} \int_{-\infty}^{\infty} \rho(\omega) e^{-T_E/T_2(\omega)}\, d\omega = \boxed{M_z^0 \sin\alpha_1 \sin^2\frac{\alpha_2}{2} e^{-T_E/T_2}}$$

*** (echo amplitude, ignoring freq dependence of T_2)

* $90^\circ_{y'} - \tau - 90^\circ_{y'}$	$S_1(t) = \frac{1}{2} \int_{-\infty}^{\infty} \rho(\omega) e^{-t/T_2} e^{-i\omega(t-T_E)}\, d\omega$
$90^\circ_{y'} - \tau - 180^\circ_{y'}$	$S_2(t) =$ no ½ factor
$90^\circ_{x'} - \tau - 180^\circ_{y'}$	$S_3(t) -$ multiply by i → add $\pi/2$ phase

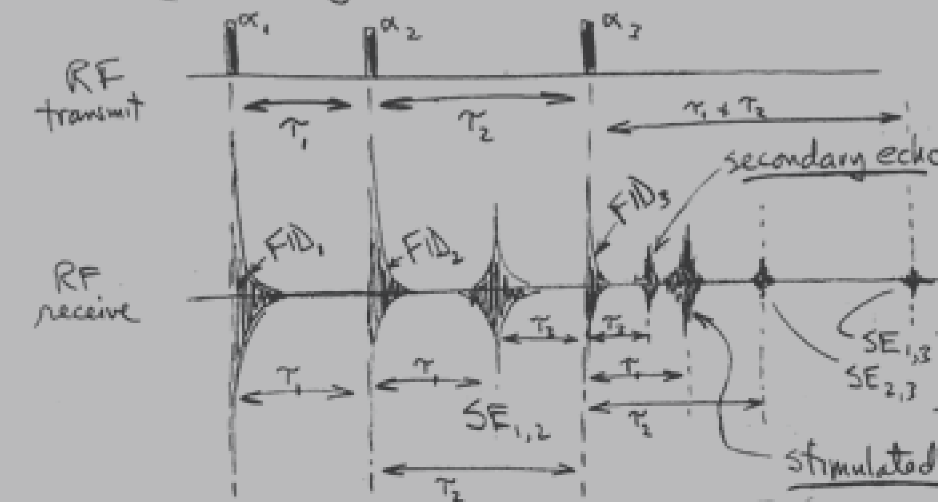
→ etc for A_E … like ↑

ECHO TRAINS – spin-echo trains

– it's (too) easy to make echos …

$$E_n = \frac{3^{(n-1)} - 1}{2}$$

echos after end of n^{th} pulse
3 RF's → 4 echos (here)
6 RF's → 121 echos (!!)



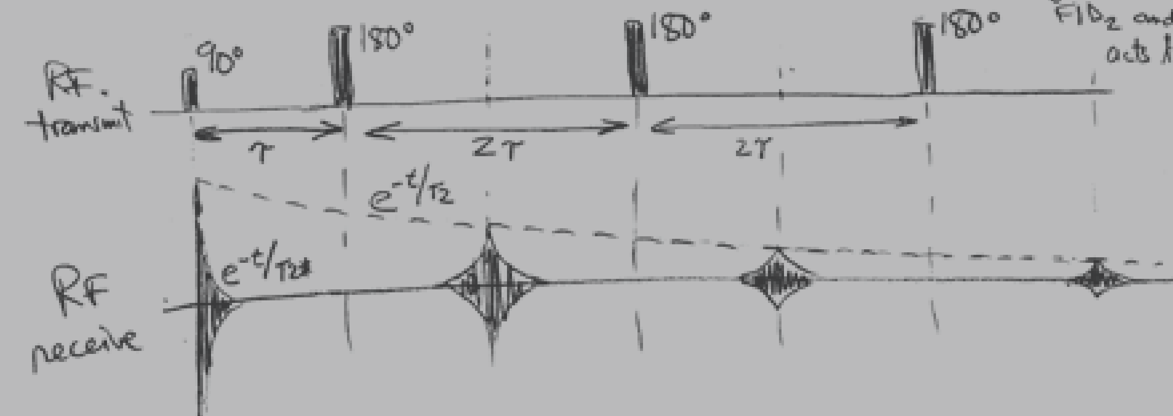
secondary echo: [SE$_{1,2}$ acts like RF pulse; α_3 makes an echo from it]

SE$_{1,3}$, SE$_{2,3}$] – two more conventional two pulse spin echoes

stimulated echo: combined effect of 3

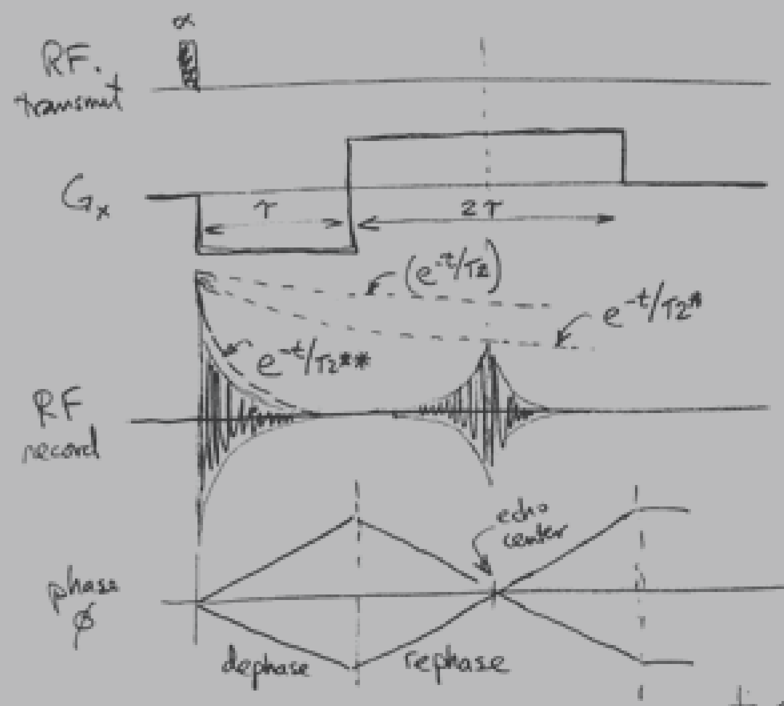
[α_1: $M_L \rightarrow M_T$
α_2: leftover M_T flipped to M_L ("saved")
α_3: flip saved $M_L \rightarrow M_T$ which can then begin to cancel delays (after being held "in limbo" between FID$_2$ and FID$_3$); acts like 2-pulse echo]

– a useful multi-echo sequence (CPMG) is a 90° followed by 180° at 2τ spacing



– typically, 90° and 180° applied in different axes (x', then y', y', y' …) which reduces phase errors due to imperfect 180° pulses (since slightly-off rotation around y' affects phase less)

GRADIENT ECHOES — T2*, GE chains



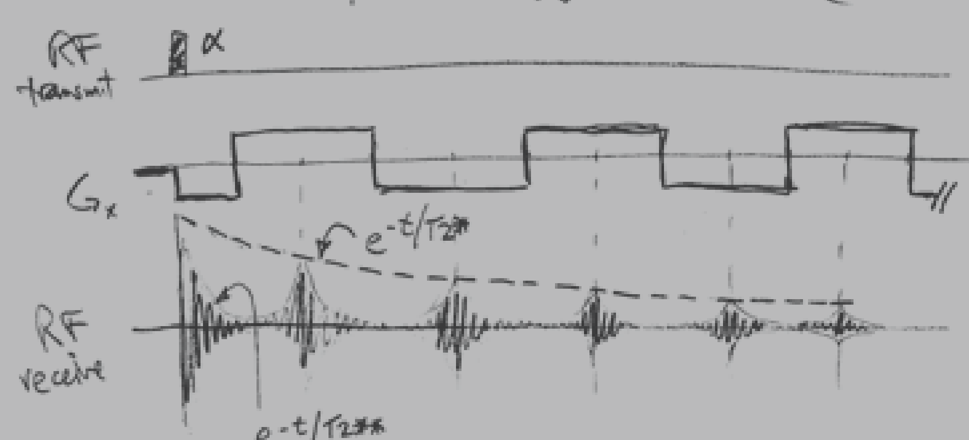
- initial negative gradient dephases spins
- after $t = \tau$ of positive gradient, spins rephase
- does not correct for T2* inhomogeneities so echo amplitude is $\boxed{A_E = e^{-t/T_2^*}}$
- the initial "FID" is not "free" since it is being actively de-phased by gradient, so FID decay

$\frac{1}{T_2} < \frac{1}{T_2^*} < \frac{1}{T_{2**}} \leftarrow \boxed{A_E = e^{-t/T_{2**}}}$

- key difference between spin-echo (SE) and gradient echo (GE) is that B_0 inhomogeneities not corrected

→ hence, echoes are T2*-weighted, not T2-weighted ⟹ more susceptible to inhomogeneities

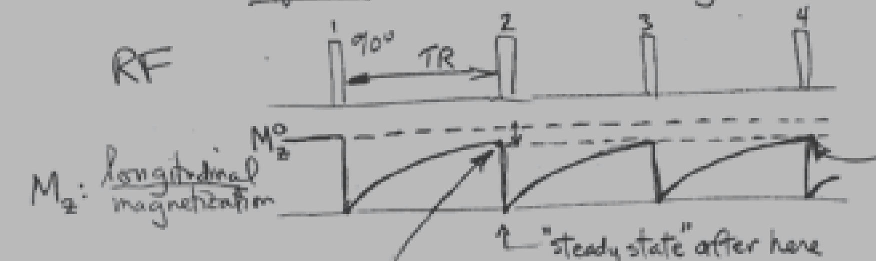
- echo trains possible w/ gradient echo (CPMG-like)



- the faster the gradients are switched, the more echoes you get
- EPI hardware ⟹ 64 echoes

IMAGE CONTRAST — T1 saturation-recovery (no echo, just FID)

- contrast (PD, T1, T2, T2*) depends on magnetization not getting back to equilibrium, and then differences in how far away each tissue type is at measurement time

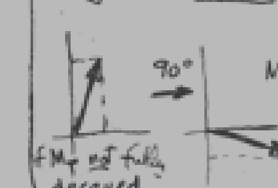


$M_z^0(1-e^{-TR/T_1}) + [\text{ignored}]$

- simple saturation/recovery w/ no echo
- initial conditions: $\left[\begin{array}{l} M_z \text{ before first pulse} = M_z^0 \\ M_z = 0 \text{ immed. after first pulse (i.e., 90° pulse)} \end{array}\right.$
- from Bloch eq, M_z just before second pulse:

$$\underbrace{M_{z'}^{(n)}(0_-)}_{M_L \text{ before current pulse}} = \underbrace{M_z^{(0)}\left(1-e^{-TR/T_1}\right)}_{M_L \text{ "regrowth-from-zero" term}} + \underbrace{M_z^{(n-1)}(0_+)e^{-TR/T_1}}_{M_L \text{ "left-immed.-after-pulse" term (N.B. decaying)}}$$

assume this is 0 because we assume that M_{xy} (transverse) completely decayed so that a 90° pulse doesn't generate any initial longitudinal



- given $\left[\begin{array}{l} (1)\ 90° \text{ pulse} \\ (2) \text{ no } M_{xy} \text{ left} \end{array}\right] \rightarrow$ pure tip: $M_{xy} = M_z$

- tip existing mag

$$* \boxed{\underbrace{M_{z'}^{(n)}(0_-)}_{\text{longitudinal mag just before pulse}} = \underbrace{M_{x'y'}(0_+)}_{\text{transverse we can record after pulse}} = \underbrace{M_z^0\left(1-e^{-TR/T_1}\right)}_{\text{transverse mag depends on T1!}}}$$

- that is, the not-completely-regrown longitudinal magnetization, which depends on T1, but which we cannot record, is completely converted to recordable transverse magnetization

assume immediate recording of signal

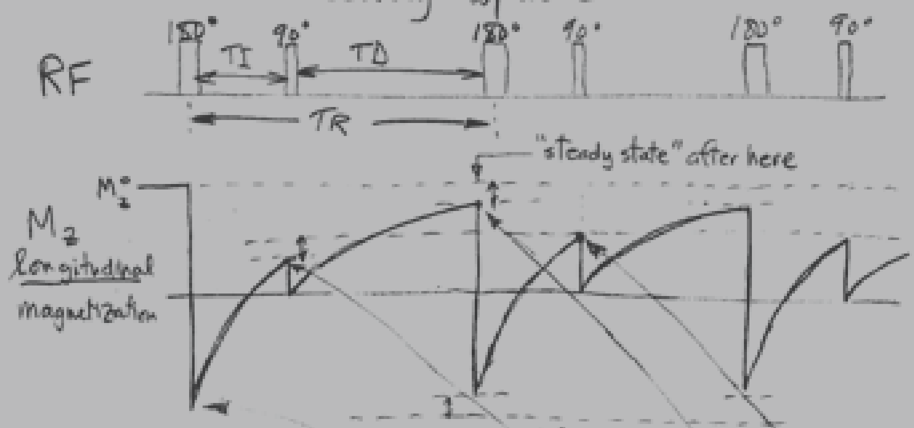
$$I(\mathbf{r}) = \underbrace{C}_{\text{recon const.}} \underbrace{\rho(\mathbf{r})}_{\text{spectral dens}} \left(1-e^{-TR/T_1(\mathbf{r})}\right)$$

spectral dens $\rho(\mathbf{r})$ ≈ p. density; underlies equilib. M_z^0

Contrast-2

IMAGE CONTRAST   IR (still just saturation-recovery — no echo)

- inversion recovery w/ no echo



- 180 deg pulse reverses longitudinal magnetization

$$M_{z'}^{(0)} = -M_z^\circ$$

- recovery to end of first TI from long. part of Bloch eq.

$$M_{z'} = M_z^\circ\left(1 - \underbrace{2e^{-TI/T1}}_{\text{from 180°, since } -M_z \text{ in second Bloch term}}\right)$$ ⟹ flipped into transverse by second pulse (1st 90)

- longitudinal then regrows from zero (from first Bloch term only)

$$M_{z'} = M_z^\circ\left(1 - e^{-(TR-TI)/T1}\right)$$

- after second 180°, just change sign again

$$M_{z'} = -M_z^\circ\left(1 - e^{-(TR-TI)/T1}\right)$$

- apply relaxation eq. again

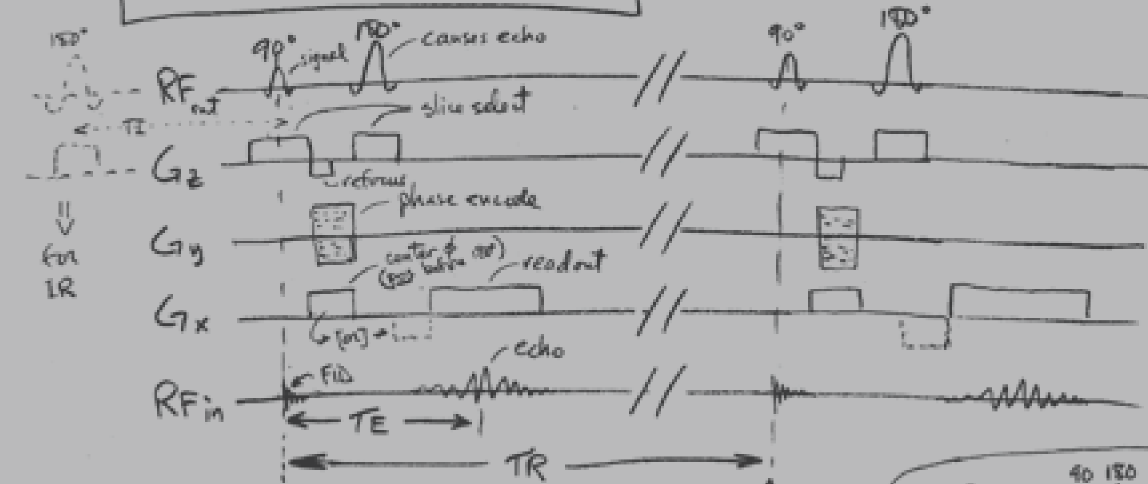
$$M_{z'} = M_z^\circ\left(1 - e^{-TI/T1}\right) - M_z^\circ\left(1 - e^{-(TR-TI)/T1}\right)e^{-TI/T1}$$

*
$$\boxed{M_{z'} = M_z^\circ\left(1 - 2e^{-TI/T1} + e^{-TR/T1}\right)}$$

↳ this is magnetization flipped to transverse, therefore made recordable

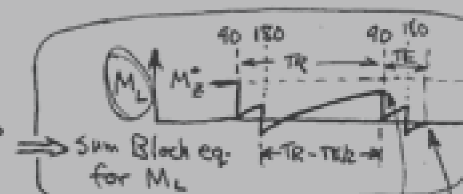
Contrast-3

IMAGE CONTRAST   SE, IR-SE



[N.B. centering G_x lobe also causes gradient echo]

- steady state mag (2nd TR) just before 90° ⟹ sum Bloch eq. for M_L



$$M_{z'}(0_-) = M_z^\circ\left(1 - 2e^{-(TR-TE/2)/T1} + e^{-TR/T1}\right)$$

- the echo signal (M_T) unlike in simple saturation-recovery FID has an additional delay before it is recorded, so we have to take account of transverse mag relaxation

$$A_E = M_z^\circ\left(1 - 2e^{-\overset{\to TR}{(TR-TE/2)}/T1} + e^{-TR/T1}\right)e^{-TE/T2}$$ from spin echo equation half

- if we assume TE much less than TR, then we can simplify:
  ↳ TR−TE ≈ TR

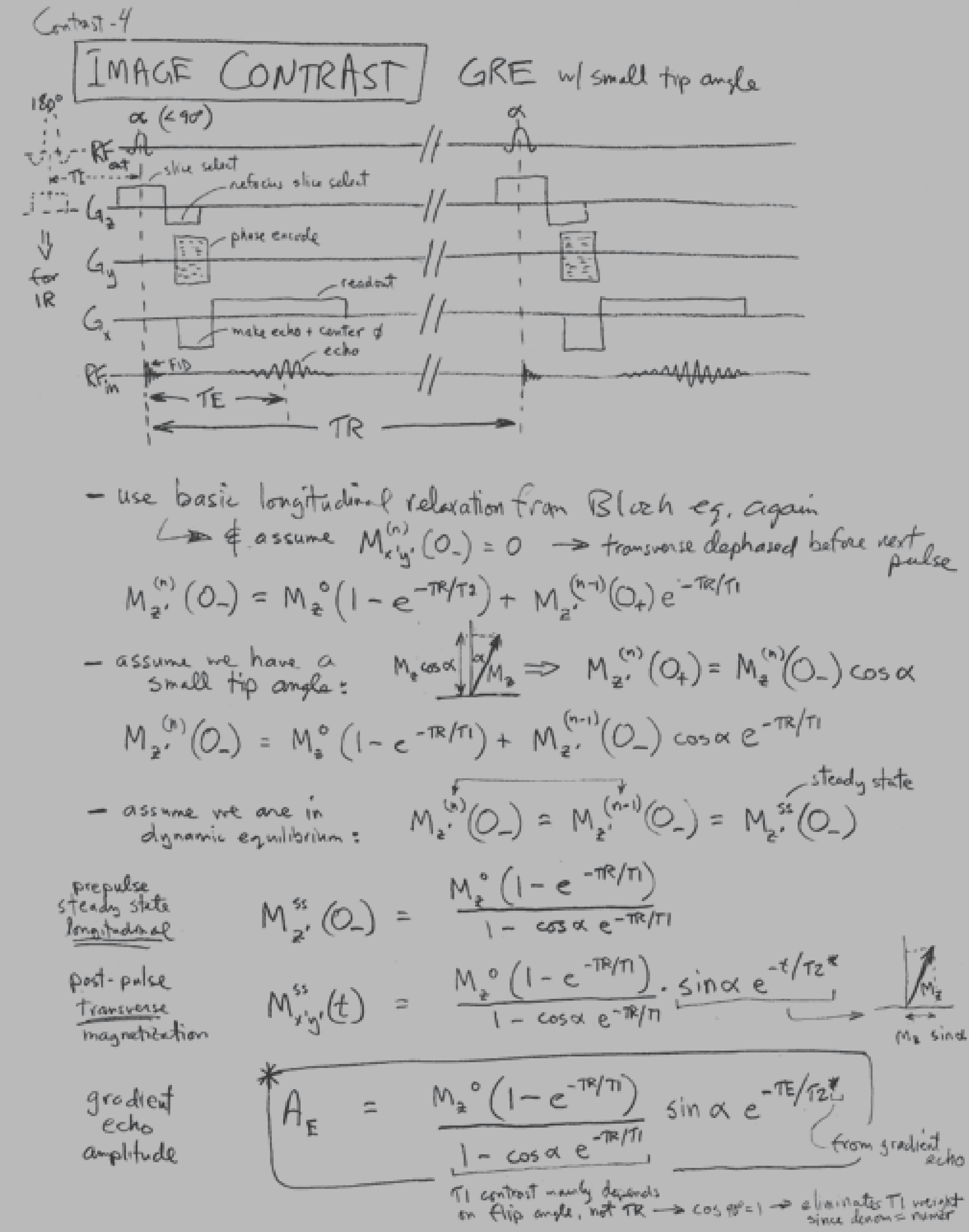
*
$$\boxed{\underbrace{A_E}_{\text{amplitude echo}} = \underbrace{M_z^\circ}_{\text{proton density}}\underbrace{\left(1 - e^{-TR/T1}\right)}_{\text{TR controls T1 contrast}}\underbrace{e^{-TE/T2}}_{\text{TE controls T2 contrast}}}$$

- similar equation for SE-IR

*
$$\boxed{A_E = M_z^\circ\left(1 - 2e^{-TI/T1} + e^{-TR/T1}\right)e^{-TE/T2}}$$

In contrast, MRI images are all in greyscale. The data retrieved during a brain scan is first organized in the so called K-space, a greyscale raster containing all the coordinates of the collected data and from this a 2 dimensional black and white image is constructed.

Снимки мозга, сделанные с помощью магнитно-резонансной томографии (МРТ), напротив, делаются в серой шкале. Данные полученные во время сканирования мозга сначала организовываются в так называемое к-пространство, точечную матрицу, содержащую все координаты собранных данных, на основе которой создаётся двухмерное черно-белое изображение.



A finished MRI image might be colourized to enhance clarity or to make for a more spectacular presentation, but these colours are not directly based on information collected during the MRI scan, and are subjected to a fair amount of speculation. Depending on what a scientist wishes to show with the image, the colouring can be used to enhance certain areas of interest, whilst making other areas seem of less significance. These images, with strong catchy colours look very convincing and scientific, but we have to remember that the colours only tell us as much as we are willing to trust whomever have been digitally editing the images for the presentation.

Завершенный снимок МРТ может быть разукрашен для большей наглядности, но эти цвета не основываются на полученных в результате сканирования данных, и поэтому могут быть   подвергнуты определенной доли спекуляции. В зависимости от того, что ученый хочет показать этим снимком, расцветка может подчеркнуть интересные для данного случая зоны, делая, таким образом, остальные части снимка менее важными. Эти снимки броской окраски выглядят очень убедительно и научно, но нам нужно помнить, что эти цвета говорят нам ровно столько, насколько мы доверяем тому, кто делал цифровую окраску снимков, готовя их к презентации.

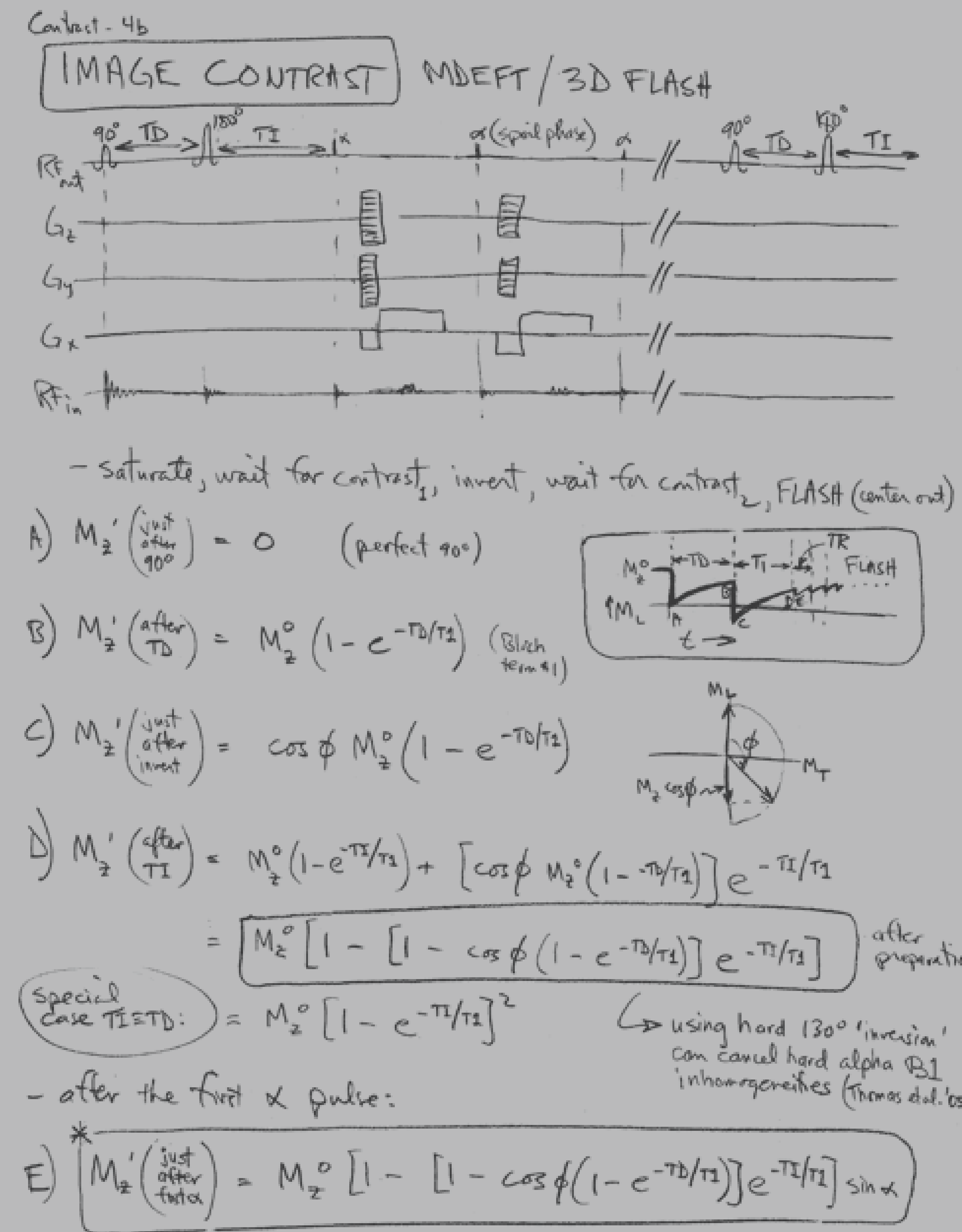

Contrast - 4b

IMAGE CONTRAST MDEFT / 3D FLASH

- saturate, wait for contrast$_1$, invert, wait for contrast$_2$, FLASH (center out)

A) $M_z'\left(\text{just after } 90°\right) = 0$ (perfect 90°)

B) $M_z'\left(\text{after TD}\right) = M_z^0\left(1 - e^{-TD/T1}\right)$ (Bloch term #1)

C) $M_z'\left(\text{just after invert}\right) = \cos\phi\, M_z^0\left(1 - e^{-TD/T1}\right)$

D) $M_z'\left(\text{after TI}\right) = M_z^0\left(1 - e^{-TI/T1}\right) + \left[\cos\phi\, M_z^0\left(1 - e^{-TD/T1}\right)\right] e^{-TI/T1}$

$$= \boxed{M_z^0\left[1 - \left[1 - \cos\phi\left(1 - e^{-TD/T1}\right)\right] e^{-TI/T1}\right]}$$ after preparation

Special case TI=TD: $= M_z^0\left[1 - e^{-TI/T1}\right]^2$

using hard 180° 'inversion' can cancel hard alpha B1 inhomogeneities (Thomas et al. '05)

- after the first α pulse:

E) * $\boxed{M_z'\left(\text{just after first } \alpha\right) = M_z^0\left[1 - \left[1 - \cos\phi\left(1 - e^{-TD/T1}\right)\right] e^{-TI/T1}\right] \sin\alpha}$

CONTRAST-TO-NOISE

Haacke

tissue	T1 (1.5T)	T2	PD
GM	950	100	0.80
WM	600	80	0.65
CSF	4500	2200	1.00 water
blood	1200	100-200	

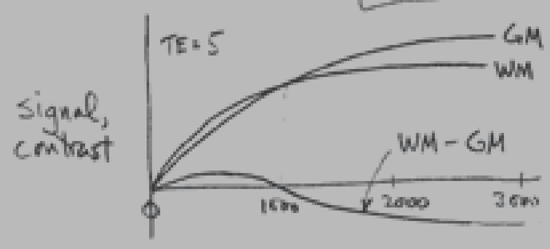
Lauterbur

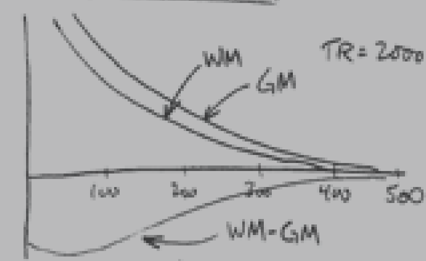
tissue	T1	T2	PD
GM	760	77	0.69
WM	510	67	0.61
CSF	2650	280	1.00

- "contrast" defined as difference: $C_{AB} \equiv S_A - S_B$
- contrast-to-noise ratio:

$$CNR_{AB} \equiv \frac{C_{AB}}{\sigma_n} = \frac{S_A - S_B}{\sigma_n} = SNR_A - SNR_B$$

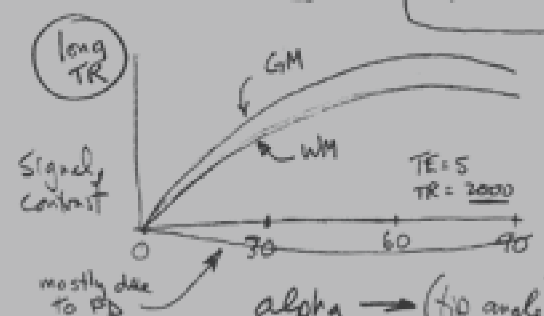
- spin-echo: $\boxed{A_E = M_z^\circ \left(1 - e^{-TR/T1}\right) e^{-TE/T2}}$

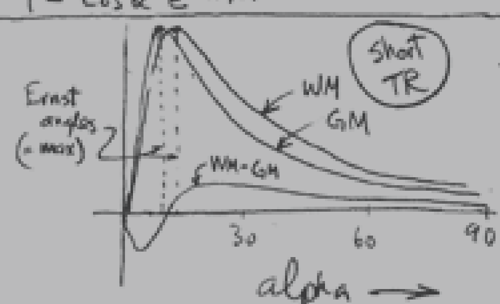




- gradient echo: $\boxed{A_E = \frac{M_z^\circ \left(1 - e^{-TR/T1}\right)}{1 - \cos\alpha \, e^{-TR/T1}} \sin\alpha \, e^{-TE/T2^*}}$

$T2^*_{GM} = 50$
$T2^*_{WM} = 40$





- general rules: spin-echo, long TR GE

proton-density weighted	TR ↑↑ (no T1 diffs)	TE ↓↓ (no T2 diffs)
T1-weighted	TR ≈ T1 (big T1 diffs)	TE ↓↓ (no T2 diffs)
T2-weighted	TR ↑↑ (no T1 diffs)	TE ≈ T2 (big T2 diffs)

SIGNAL-TO-NOISE S/N

- generalized dependence of SNR on 3D imaging parameters

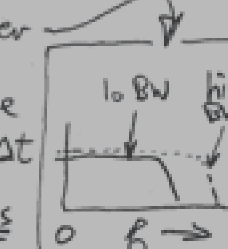
$$\boxed{SNR/voxel \propto \underbrace{\Delta x \Delta y \Delta z}_{\text{voxel size (of same number)}} \sqrt{\underbrace{N_{acq}}_{\text{num repeats}} \underbrace{N_x N_y N_z}_{\text{number of voxels (of same size)}} \underbrace{\Delta t}_{\text{read timestep}}}}$$

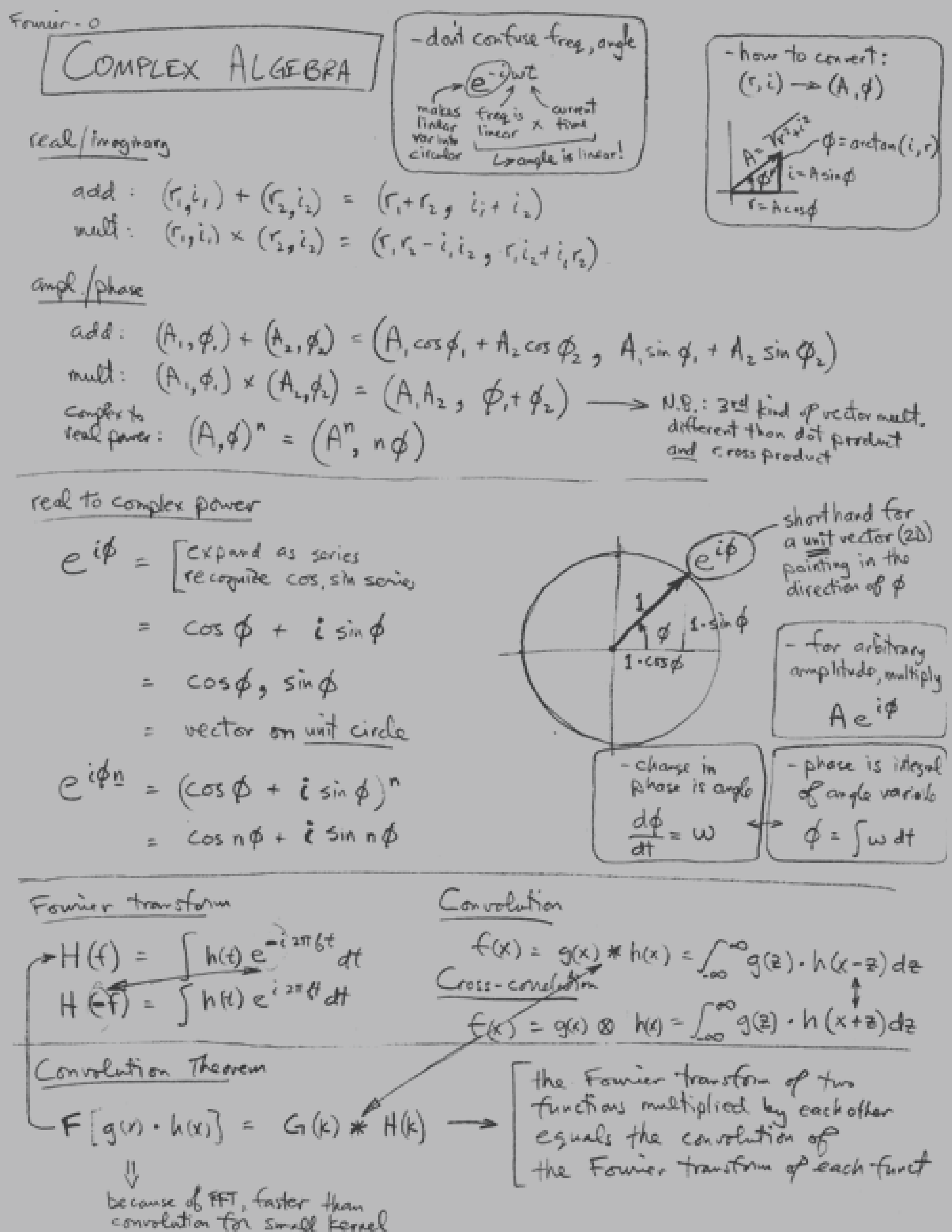
- size (volume) of voxels (with the number of voxels held constant), linear effect on S/N
  - ↳ e.g., $\boxed{3 \times 3 \times 3\text{ mm} \rightarrow 4 \times 4 \times 4\text{ mm}} \rightarrow \frac{64}{27} = 2.37$ times better S/N
- more voxels (with size of voxels, Δt per read step constant) $\sqrt{n}$ effect on S/N
  - ↳ e.g., $\boxed{64 \times 64 \rightarrow 128 \times 128} \rightarrow \frac{\sqrt{128 \cdot 128}}{\sqrt{64 \cdot 64}} = 2$ times better S/N
- # acquisitions $\sqrt{n}$ better S/N
  - ↳ e.g. $\boxed{1\text{ acq} \rightarrow 2\text{ acq}} \rightarrow \frac{\sqrt{2}}{\sqrt{1}} = 1.41$ times better S/N

- longer timestep during readout, $\sqrt{\Delta t}$ better S/N

$\Delta t = \frac{1}{BW_{read}}$, digitization timestep during echo acquisition

- BW_{read} determined by cutoff freq, analog lowpass filter
- Δt controls BW because low pass cutoff has to be set higher for smaller (higher freq-detecting) Δt
- must filter out freq's $> f_{max} = \frac{1}{2\Delta t}$ because they alias





What comes out of the MRI scanner is always limited to greyscale.

Всё, что мы получаем в результате МРТ сканирования всегда ограничено серой шкалой.

Fourier transform (1)

ft is a phase angle
$f \cdot t = ft$
cyc/sec · sec = cyc

for one f

$$\underbrace{H(f)}_{\text{complex freq. domain}} = \int_{t=-\infty}^{t=\infty} \underbrace{h(t)}_{\text{complex time domain}} \cdot \underbrace{e^{-i\,2\pi f t}}_{\text{complex periodic}} \, \underbrace{dt}_{\text{real}}$$

real · real · complex multiply · ang. freq (rad/sec) ω · lin freq (cyc/sec)

sum across t (complex add)

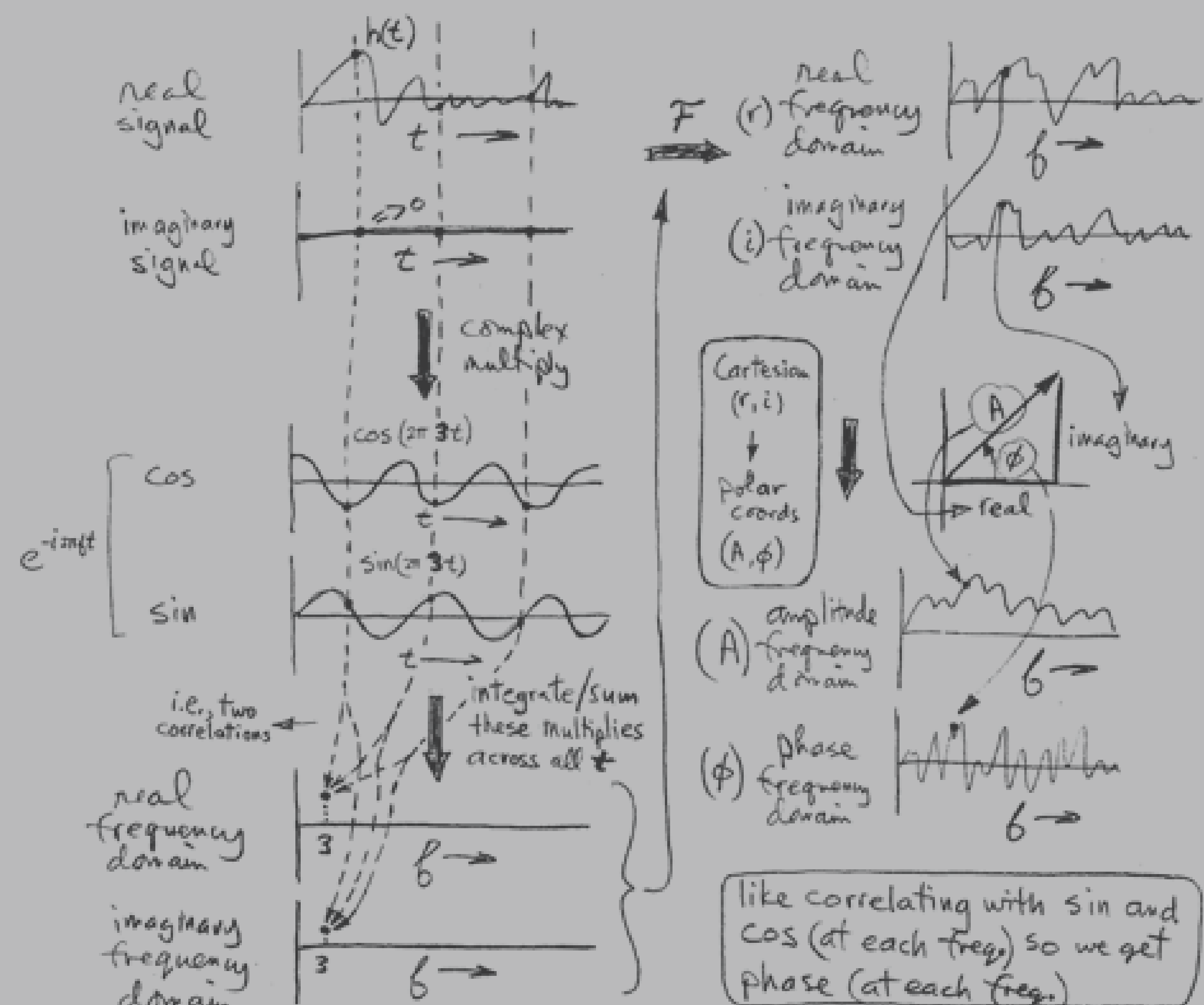
Fourier integral terms written out

$h_r(t)\cos(2\pi f t) + h_i(t)\sin(2\pi f t)$
$-h_r(t)\sin(2\pi f t) + h_i(t)\cos(2\pi f t)$

Fourier integral w/ real signal

$h_r(t)\cos(2\pi f t)$,
$-h_r(t)\sin(2\pi f t)$

- How to calculate $H(f)$ for one f $(f=3)$:

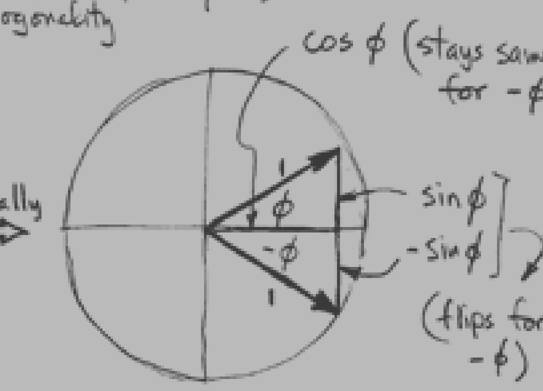


Fourier transform (1b)

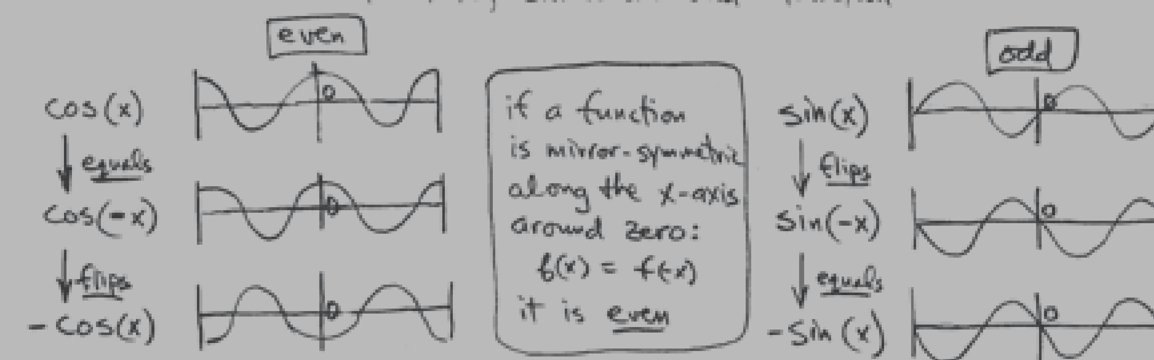
- neg complex exponents/freq
- orthogonality

$$e^{i\phi} = \boxed{\cos\phi + i\sin\phi}$$
$$e^{-i\phi} = e^{i(-\phi)}$$
$$= \cos(-\phi) + i\sin(-\phi)$$
no effect (see below) · move outside
$$= \boxed{\cos\phi - i\sin\phi}$$

graphically ⟹



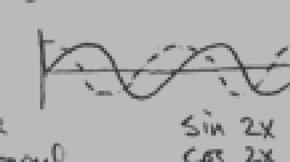
- **cos** is an "even" function, **sin** is an "odd" function



An orthogonal decomposition

- think of discretely sampled $\sin(fx)$, $\cos(fx)$ as vectors
- $\mathrm{Corr}(\vec{v}_1, \vec{v}_2) \equiv$ projection of v_1 onto v_2 $\equiv \vec{v}_1 \cdot \vec{v}_2$

$\mathrm{Corr}(\cos f_1 x, \sin f_1 x) = 0$
= sin & cos of same frequency are orthogonal

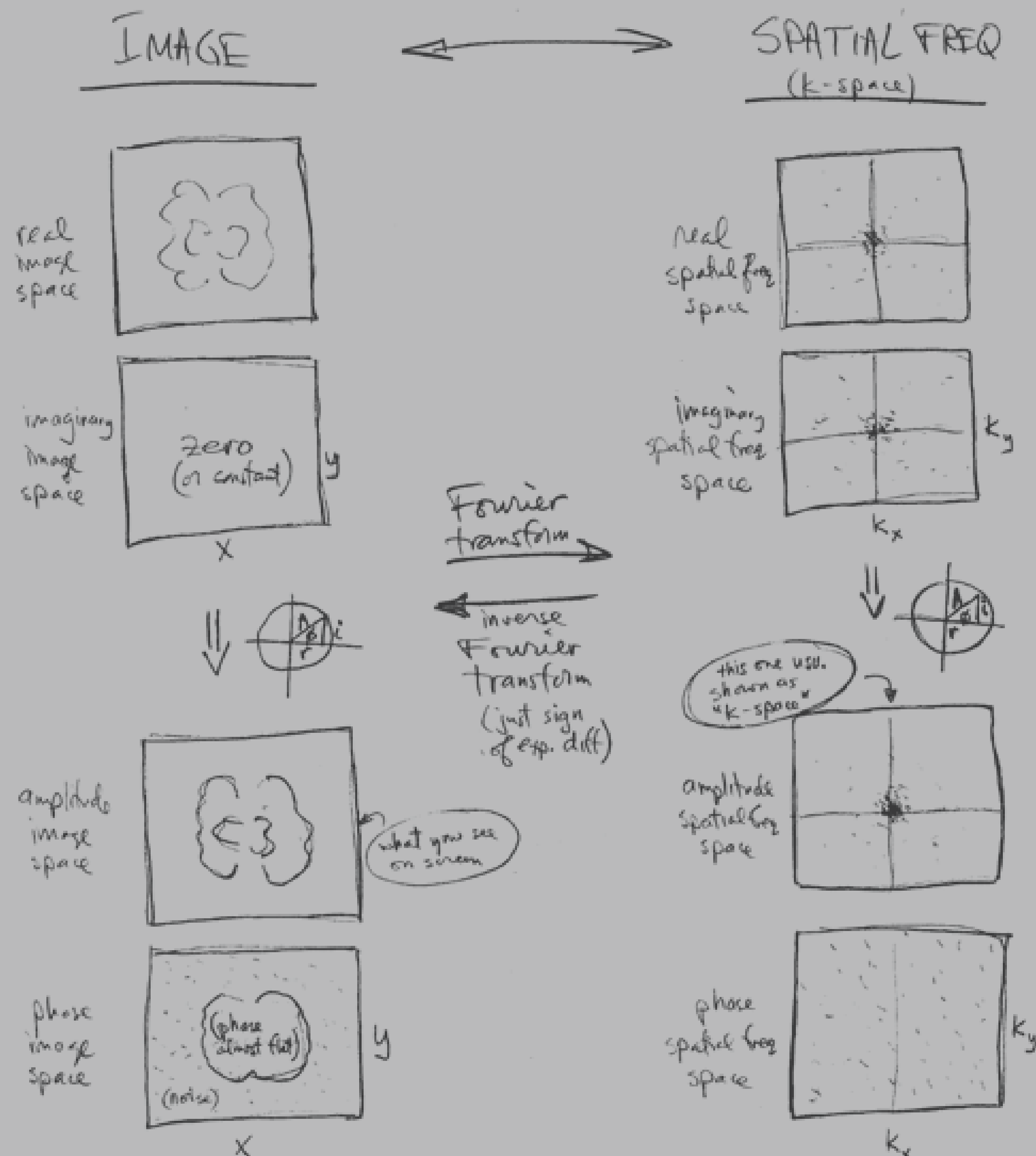


$\mathrm{Corr}(\sin f_1 x, \sin f_2 x) = 0$
= different integer freqs of sin (or cos) are orthogonal

$\mathrm{Corr}(\cos f_1 x, \sin f_2 x) = 0$ [as above]

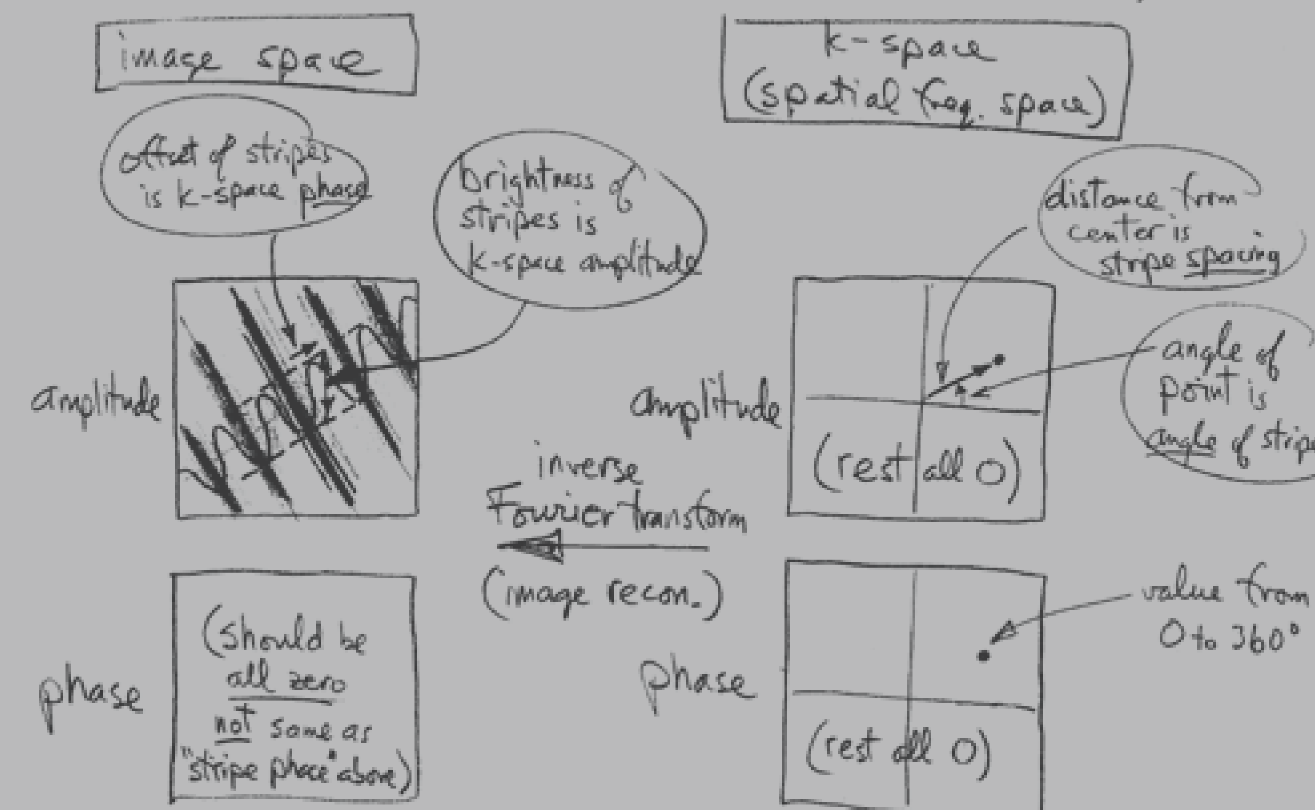
- in the continuous case, orthogonal functions defined as: $\int_{x=lo}^{x=hi} f(x)\, g(x)\, dx = 0$

FOURIER TRANSFORM OF IMAGE (2)



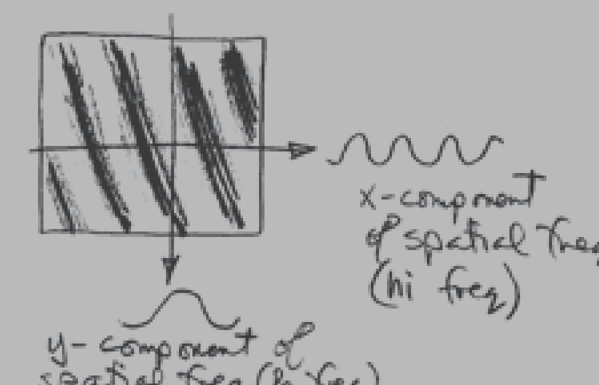
FOURIER TRANSFORM OF AN IMAGE (3)

– what a single k-space point looks like in image space (polar coordinates A, φ instead of r, i)



– Cartesian dimension of k-space — x- and y- spatial freq

N.B.: each dimension of spatial freq. space (k-space) from correlation w/ sin + cos — don't confuse k_x, k_y w/ sin, cos!



N.B. – increasing one 1D component increases the spatial freq of the 2D wave and rotates it

GRADIENT COILS

- gradient coils for x, y, z generate approximately* a linear gradient in the strength of the z-component of the magnetic field B_z

all effects of gradients discussed before were in z-direction (parallel to B_0)

- for example, the x gradient coil induces a ramp in z-component of the magnetic field when moving in the x-direction:

$$B_{G,z} = G_x x$$

*- since a pure linear gradient of $B_{G,z}$ in only the x, y, or z directions is not possible according to Maxwell equations, each gradient coil also induces a magnetic field that has components in the x- and y-direction ($B_{G,x}$ and $B_{G,y}$)

- the other magnetic field components are (2 to 3 orders of mag. less) usu. ignored because they are so small relative to $B_{G,z}$, since $B_{G,z}$ is added to B_0, and since B_0 is much stronger than $B_{G,z}$, $B_{G,y}$, & $B_{G,z}$

- since standard reconstruction methods assume the existence of "non-Maxwellian" gradient fields, spatial distortion is introduced

- the Maxwellian terms $B_{G,x}$ $B_{G,y}$ are known; can be included in the recon. process

e.g. extra phase in z-dir ⊥ to x grad

$$\Delta\phi\, G_x(\vec{x}) \approx \frac{-\gamma z^2 G_0^2 t}{2B_0}$$

There is not one person that understands the whole process from the actual scan to the interpretation of the results. In this, engineers, radiologists, data analysts, neuro-physicists and/or neuro-psychologists are all involved. Translations between several different types of languages are required.

Нет ни одного человека, кто понимал бы весь процесс от непосредственно сканирования до расшифровки результатов. В этом процессе участвуют инженеры, радиологи, аналитики данных, нейрофизики и/или нейропсихологи. Для этого требуются переводы между разными типами языков.

SLICE SELECTION (G_z)

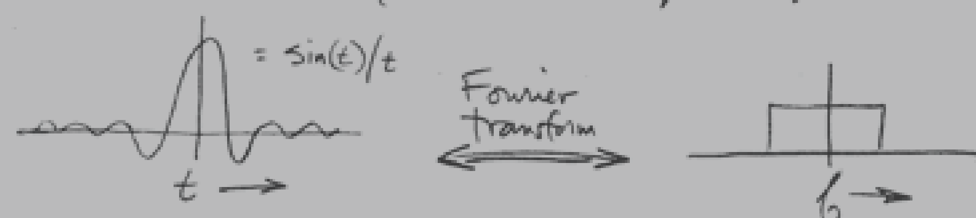
- slice select gradient on during RF stim

$f = \gamma(B_0 + B_{G,z})$, with $\frac{\gamma}{2\pi}$ and $G_z z$

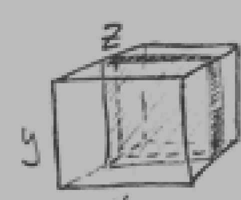
- protons here can only be excited by a narrow band of radio frequencies

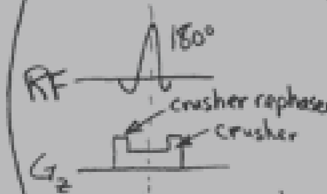
- to apply a pulse containing a narrow band of frequencies, we use a sinc pulse envelope (Fourier transform of a narrow freq. band)

in practice, Gaussian pulse envelope good, too



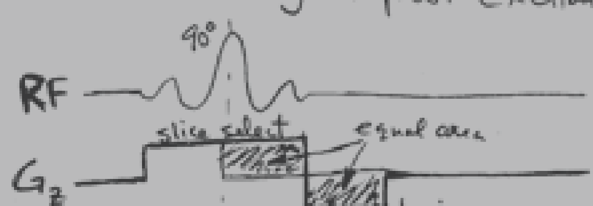
- this excites protons in a narrow slab





- 180° pulses don't need rephasing (self-rephasing)
- crusher lobes added to kill spurious transverse

- since the slice-selection gradient introduces (space-dependent) phase shifts (see freq encode) these have to be removed by a post-excitation rephasing z-gradient



- approximation from assuming tip occurs instantaneously in middle
- valid for small tip: 90° → 52%
- in practice: adjust to max, use crusher to kill spurious transverse on 180°

PULSES FOR SLICE SELECTION

Fourier approach details

- Fourier transform approach to slice-selective pulse (linear approx. even tho tipping is non-linear)

$$\underbrace{B_1(t)}_{\text{time dependent RF stimulation (complex)}} \propto \int_{-\infty}^{\infty} \underbrace{p(f)}_{\text{frequency selection function}} e^{-i 2\pi f t} \, df$$

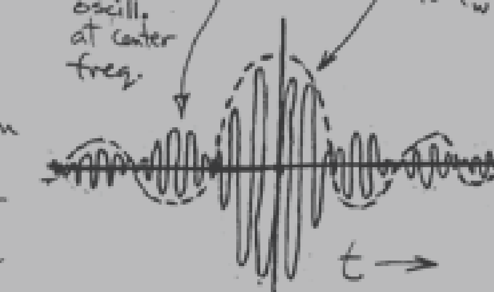
i.e., time-dependent complex (= quadrature) pulse waveform is Fourier transform of frequency spectrum of RF pulse

Solve with: $p(f)$ = frequency band:

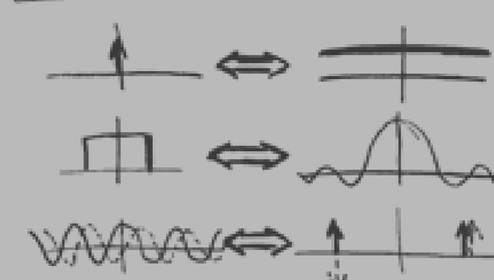
sinc envelope width inversely proportional to f_w

$$B_1(t) = \underbrace{A}_{\text{amplitude controlling flip angle}} \cdot \underbrace{f_w}_{\text{freq width (controls slice width)}} \underbrace{\operatorname{sinc}(\pi f_w t)}_{\text{sinc envelope determined by freq width, } f_w \text{ (N.B. wider } f_w \text{ is narrower sinc)}} \underbrace{e^{-i 2\pi f_c t}}_{\text{modulation (complex) at center freq., } f_c}$$

Larmor oscill. at center freq.

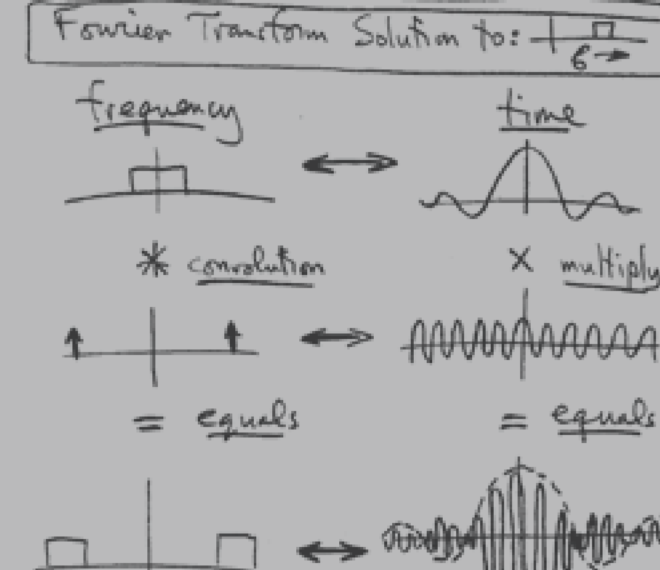


Fourier Transform Pairs, Rules



- convolution in one domain is multiplication in the other
- convolution with delta funct. impulse moves function to impulse center

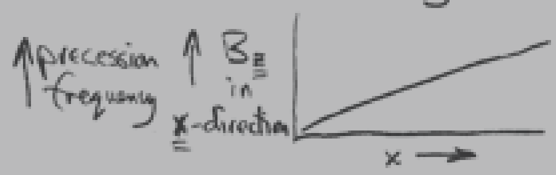
Fourier Transform Solution to:



FREQUENCY ENCODING (1)

avoid this mistaken intuition!

- frequency encode gradient (G_x) causes precession rates to vary linearly in x-direction



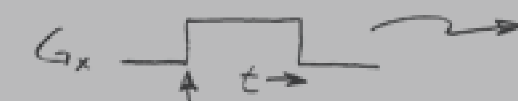
↳ correct (remember that strength of G_x causes variation of slope of B_z in x-direction)

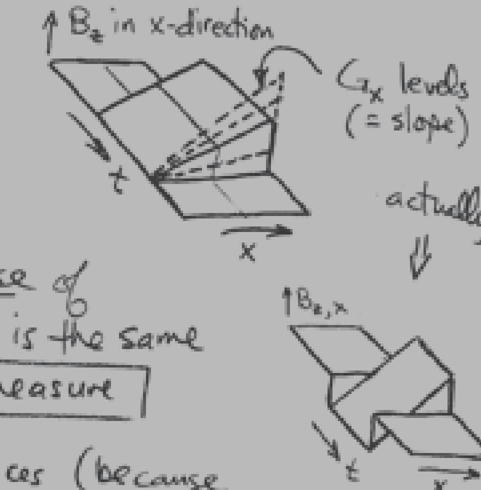
- different frequency signals are mixed together and recorded as a 1-D signal over time
  ↳ correct, but remember, we are recording the summed phase angle after de-modulation
- a Fourier transform, which converts back and forth between x-position (cf. time) and spatial frequency (cf. temporal freq) is done on signal
  ↳ correct
- spatial frequencies get confused/conflated with precession frequencies
  ↳ wrong !!
- therefore, the Fourier transform is used to convert position-dependent precession frequencies into spatial position
  ↳ wrong!! → it actually converts spatial frequencies to spatial position
  → the spatial frequency increases for each time point in the readout
  → the precession freq ramp is constant each timestep

FREQUENCY ENCODING (2)

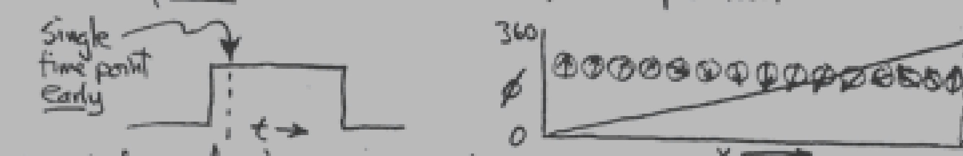
correct intuition — why phase critical

- "frequency"-encode gradient (G_x) turned on during during echo causes precession rates to immediately vary with x-position



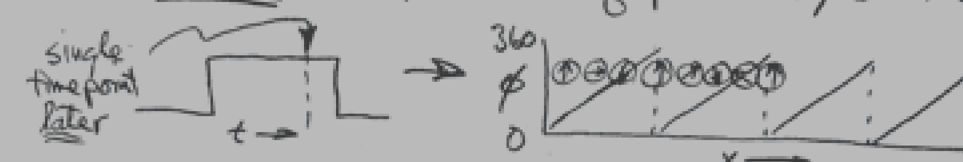


- at beginning of gradient on, the phase of signal coming from each x-position is the same
  $\boxed{\text{summed phase angle is what we measure}}$
- early after gradient on, phase advances (because of faster precession frequency) arise with greatest phase advance at largest x-position



⇒ one cycle of spatial frequency of phase angle (= low spatial freq)

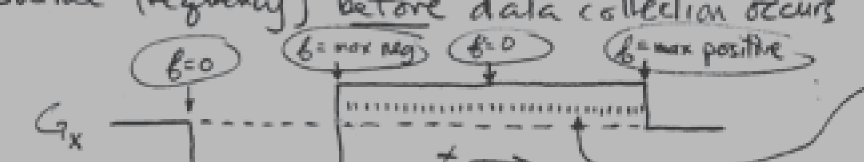
- later during gradient on, phase advances cause multiple wraparounds of phase angle across space



⇒ multiple cycles of spatial frequency of phase angle (= hi spatial freq)

- in practice, the lowest spatial frequency (= 0) occurs in the middle of the gradient on time because the phase is "wound" negatively by a preparatory gradient (to the highest negative spatial frequency) before data collection occurs

δ is spatial frequency



individual RF data samples (after demodulation)

FREQUENCY ENCODING (3) why each datapoint is 1 spatial freq

Standard Fourier transform : (Temporal freq ⟷ time)

$$\underbrace{H(f)}_{\text{freq domain}} = \int_{t=-\infty}^{t=\infty} \underbrace{h(t)}_{\text{time domain}} \cdot \underbrace{e^{-i 2\pi f t}}_{} \, \underbrace{dt}_{\text{sum across all time}}$$

time domain ↳ r, i=0

↳ [cos $2\pi f t$, sin $2\pi f t$

"k" is often used instead of "f" for the frequency variable

Imaging equation : (spatial freq. ⟷ space)

$$\underbrace{S(f)}_{} = \int_{x=-\infty}^{x=\infty} \underbrace{I(x)}_{} \cdot \underbrace{e^{-i 2\pi f x}}_{} \, \underbrace{dx}_{}$$

Sum across x of object

↓

this is done by RF coil recording sum

one data point (i.e., one spatial freq) during readout (2 components)

↓

get this single readout point by summing signal across x-position (RF coil records sum)

↓

even though variable is f, it represents one time point during readout

Signal strength at one x-position (brightness of image point)

↓

spin density (spectral dens.)

to make image, do inverse Fourier transform of recorded signal $S(f)$

cos $(2\pi f x)$, -sin $(2\pi f x)$

↓

oscillations come from readout phase wrapping, where f is single spatial freq (e.g. 5) and x goes across object

↓

$f = G_x t$, that is, spatial freq depend on amount of time gradient was on (this f increases with time!)

[FID: $G_x t$
SE: $G_x(t-TE)$

↓

don't confuse with instantaneously changed precession freqs which are constant across entire readout time (for each x position)

ALTERNATE DERIVATION (incl. effects of G_x) SIGNAL EQ

- oscillators at $\omega = \gamma B$ at each position (just x for now)

$$S(t) = m(x)\, e^{-i\phi(x)} dx$$

- by definition, freq, ω is rate of change of phase, φ

$$\frac{d\phi(x,t)}{dt} = \omega(x,t) = \gamma B(x,t) \quad \text{and integrating} \quad \phi(x,t) = \int_0^t \omega(x,t)\,dt = \gamma \int_0^t B(x,t)\,dt$$

- assuming phase initially 0, B affected by gradients

$$B(x,t) = B_0 + G_x(t) \cdot x$$

so

$$\phi(x,t) = \gamma \int_0^t B_0\,dt + \left[\gamma \int_0^t G_x(t)\,dt\right] x$$

$$= \omega_0 t + 2\pi k_x(t)\, x$$

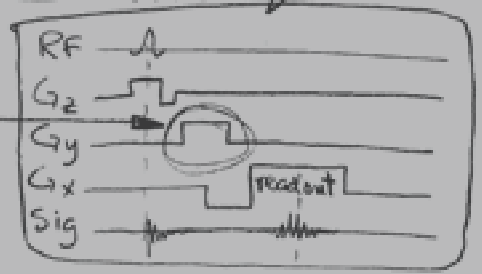
k is time integral of gradient waveform

- demodulation removes the B_0-caused carrier frequency $e^{-i\omega_0 t}$ from the first equation

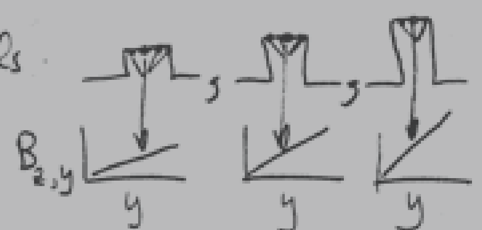
$$S(t) = \int_x \underbrace{m(x)}_{\text{amplitude of each oscillator}}\, \underbrace{e^{-i 2\pi k_x(t) x}}_{\text{gradient-controlled phase}}\, dx$$

PHASE-ENCODE GRADIENT G_y

Pulse Sequence



- turn on gradient after excitation but before readout
- different levels of G_y



- higher levels of G_y (slope of B_z in y-direction!)
  ↳ higher spatial freq. (more phase wraps) in y-direction
- phase wraps persist after phase-encode gradient off
- read-out gradient (G_x) phase wraps then add to phase-encode phase

2D Imaging Equation

fixed strength, readout time: $k_x = G_x t$

increases each TR: $k_y = G_y$

$$\underbrace{S(k_x, k_y)}_{\text{signal recorded at single time point (one readout point)}} = \underbrace{\iint_{y\ x}}_{\text{sum across x-y plane}} \underbrace{I(x,y)}_{\text{image}} \cdot \underbrace{e^{-i2\pi(k_x x + k_y y)}}_{\text{phase}} \, dx\, dy$$

signal recorded at single time point (one readout point) ⇓ complex signal (from phase-sensitive detection)

sum across x-y plane ⇓ done by RF coil

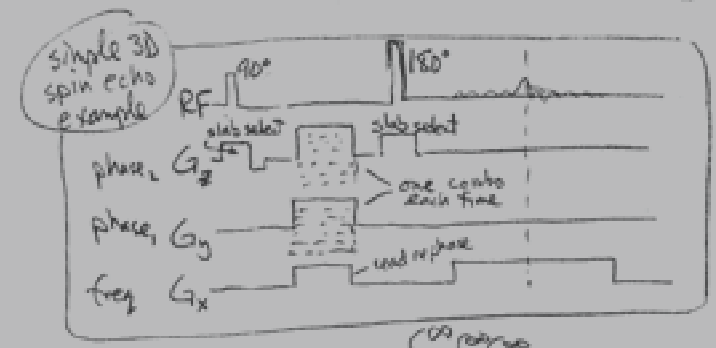
image (= strength of magnetization at each x-y pnt) ⇓ scalar (what we try to reconstruct)

phase (vector of unit length and particular angle which is function of G_x and G_y) ⇓ phase angle (of scalar magnetization!) in rotating frame, set by gradients

- ignoring relaxation, spatial frequency k_x and k_y have no "inertia" — they stay wherever the gradients last left them

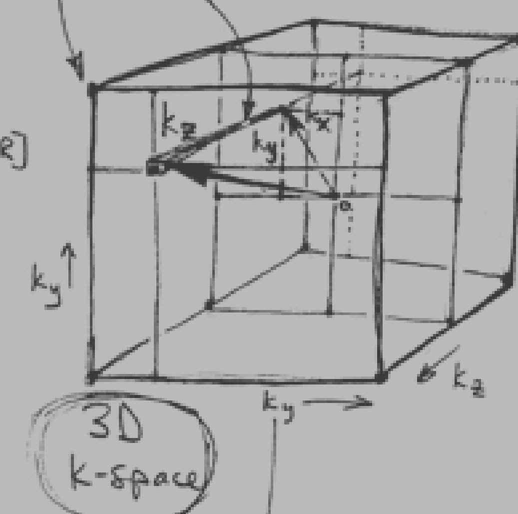
3-D IMAGING — two phase-encode gradients

- use z-gradient for 2nd phase-encoding instead of slice selection
- excitation of whole slab (slice-select is whole brain)
- simple spin echo example (in real life, usu. done with echo trains [FSE] or small flip angle to allow short TR [SPGR])



total spatial freq. greater than in any 2-D k-space point (really a sphere is optimal)

2nd phase encode add (onto orig freq encode phase)



$$\underbrace{S(k_x, k_y, k_z)}_{\text{signal recorded at single point of readout}} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x,y,z)\, e^{-i2\pi(k_x x + k_y y + k_z z)}\, dx\, dy\, dz$$

- i.e., freq-encode phase, first phase-encode phase, and second phase-encode phase all just add (= 3D rotation of phase stripes)

N.B., this ignores relaxation effects for now

- S/N much better than 2-D because each entire excited volume contributes to signal from each pulse instead of just slice
  ↳ phase stripes created throughout volume vs. slice:

one readout point in image space

PHASE & FREQ, 2D & 3D

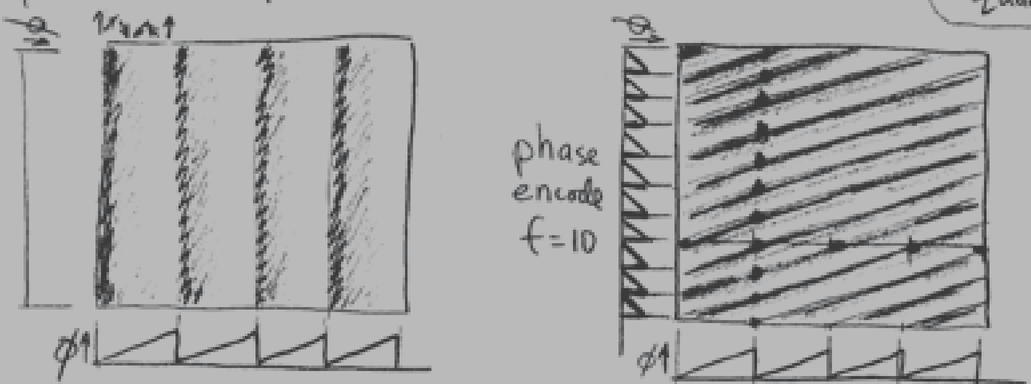
pictures of phase in image space

N.B.: stripes have sharp edges from phase wrap (not sinusoid since ϕ from 2-comp quadrature!)

- since the phase-encode gradient and the freq encode gradient both affect phase the result is a rotation of phase "stripes" when the two add

phase encode f=0

phase encode f=10



freq encode f=4

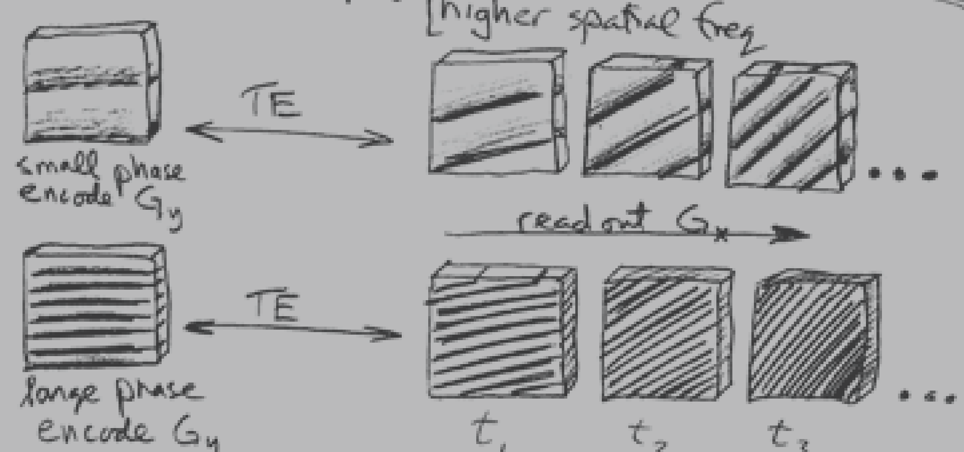
freq encode f=4 (unchanged in x-axis)

stripes here represent complex value

phase of whole image summed to one (complex) number by RF coils

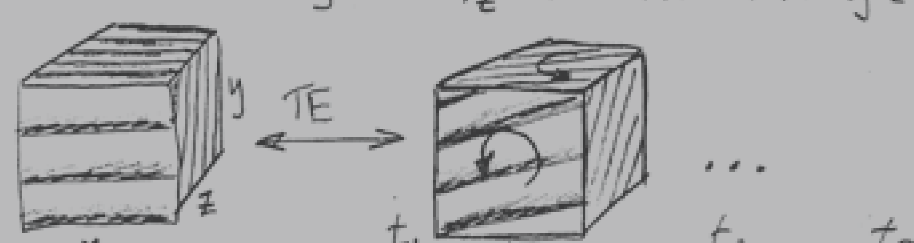
- successive read out steps: [more rotation, higher spatial freq]

e.g., after x-gradient, spins at a point might be 3 cyc ahead while after y-gradient spins at same pt 7 cyc ahead: but counting wraps in x-direction, still only 3 ahead



- 3D phase encode w/ G_y and G_z starts rotated in y-z plane

large phase encode G_z + small phase encode G_y



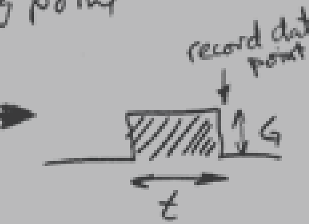
GRADIENTS MOVE K-SPACE LOCATION OF DATA POINT

- k-space (spatial-frequency space) location is set by integral of gradient over time up to recording point

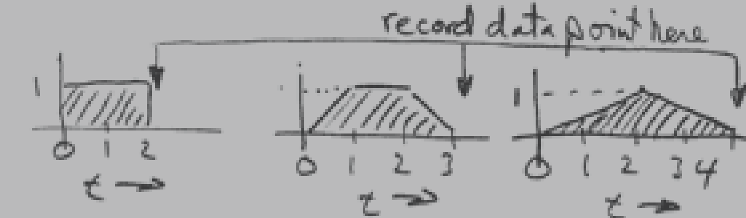
$$\underbrace{k}_{\text{spatial freq recorded at } t=\text{recordtime}} = \gamma \int_0^{t=\text{recordtime}} \underbrace{G(t)}_{\text{gradient strength as function of } t}\, dt$$

simple form of integral w/ boxcar gradient

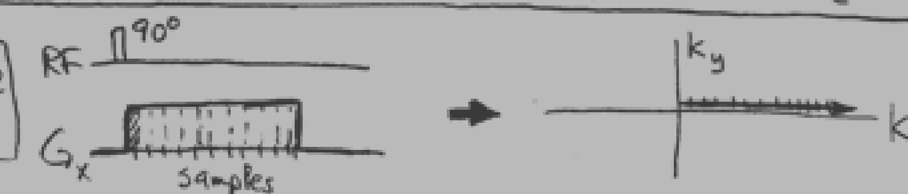
$k = Gt$ (k is area under curve)



- all of the following gradients end up at the same point in k-space:

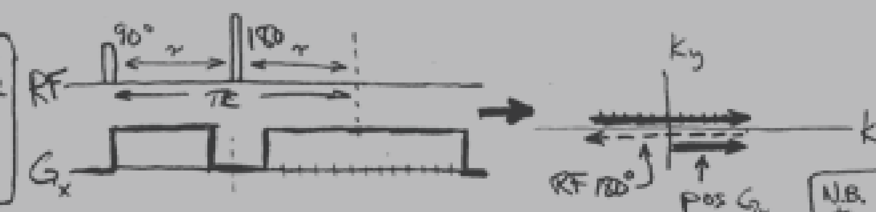


Frequency-encode FID

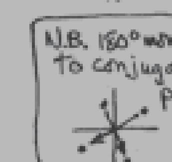


Frequency-encode gradient echo

Frequency-encode spin-echo (plus gradient echo!!)



N.B. 180° moves to conjugate pnt



Phase-encode then frequency encode gradient echo

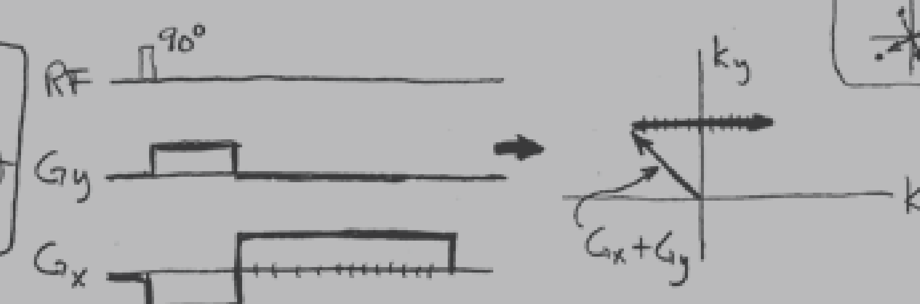


IMAGE RECONSTRUCTION

$$\underbrace{S(k_x,k_y)}_{\text{signal (complex)}} = \underbrace{\iint_{y\ x}}_{\text{RF coil sums}} \underbrace{I(x,y)}_{\text{image (real)}}\, e^{\underbrace{-i2\pi(k_x x + k_y y)}_{\text{gradient-caused phase wraps (complex)}}} dx\,dy$$

$$\underbrace{I(x,y)}_{\text{image}} = \iint_{k_y\ k_x} \underbrace{S(k_x,k_y)}_{\text{signal (complex)}}\, e^{\underbrace{+i2\pi(xk_x + yk_y)}_{\sin,\ \cos}} dk_x\,dk_y$$

[ideally → image is real
in practice → complex → use amplitude image:

$\boxed{\text{adding exponents same as multiplying two } e^{i2\pi k x}\text{'s}}$

$$= \iint_{k_y\ k_x} S(k_x,k_y)\, e^{i2\pi x k_x} e^{i2\pi y k_y} dk_x\, dk_y$$

$\boxed{\text{same as two sequential 1D FFT's (actual code)}}$

$$= \int_{k_y} \left[\int_{k_x} S(k_x,k_y)\, e^{i2\pi x k_x} dk_x \right] e^{i2\pi y k_y} dk_y$$

– in practice, finite number of samples, N and M, are collected k_x and k_y directions of k-space (integral → discrete sum)

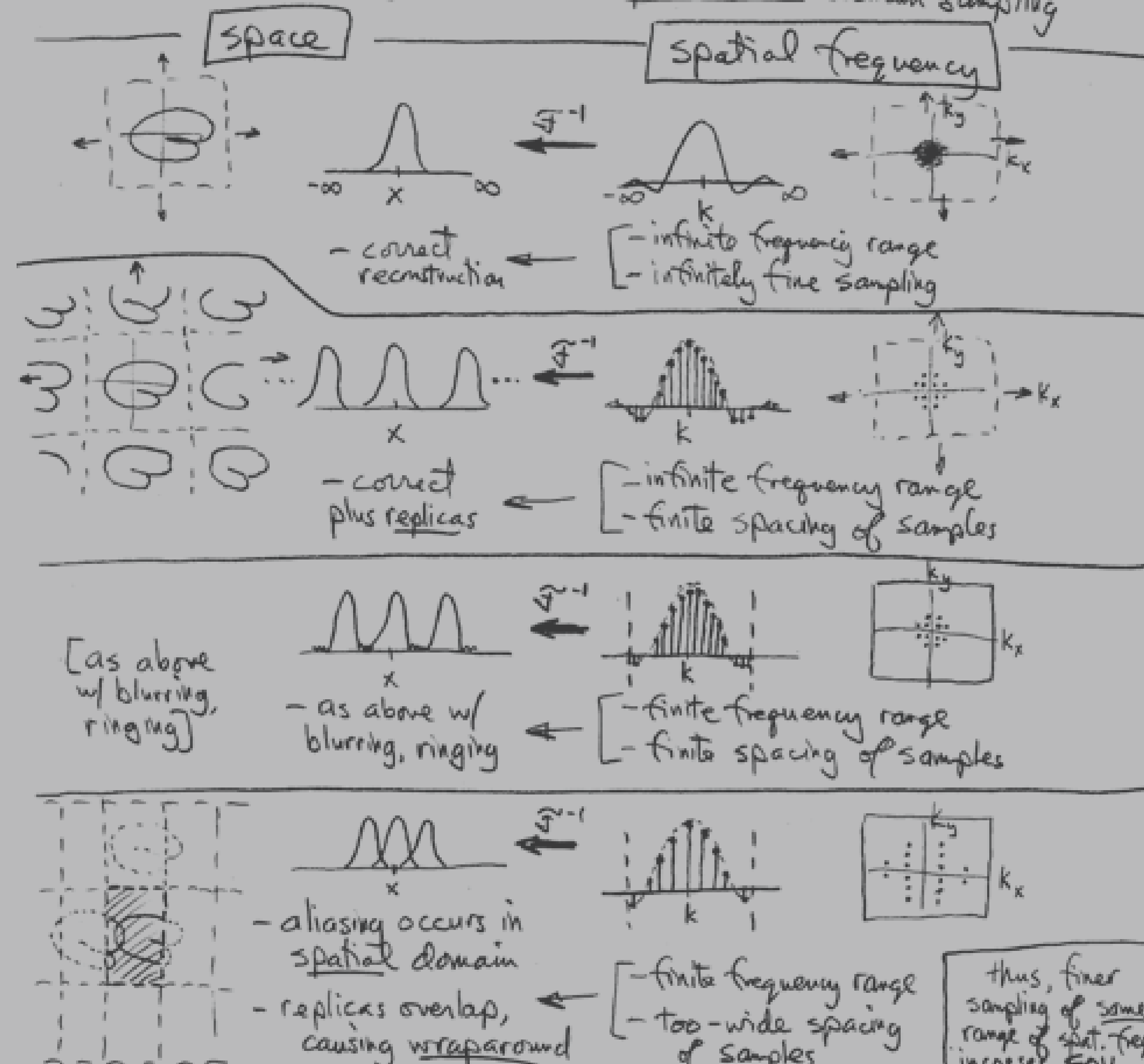
$$\underbrace{I(x,y)}_{\substack{|x| < \frac{1}{\Delta k_x} \\ |y| < \frac{1}{\Delta k_y}}} = \sum_{m=-M/2}^{M/2-1} \left[\sum_{n=-N/2}^{N/2-1} S(n,m)\, e^{i2\pi n \underbrace{\Delta k_x}_{\substack{\text{sampling}\\ \text{interval}\\ \text{in k-space}}} x} \Delta k_x \right] e^{i2\pi m \Delta k_y y} \Delta k_y$$

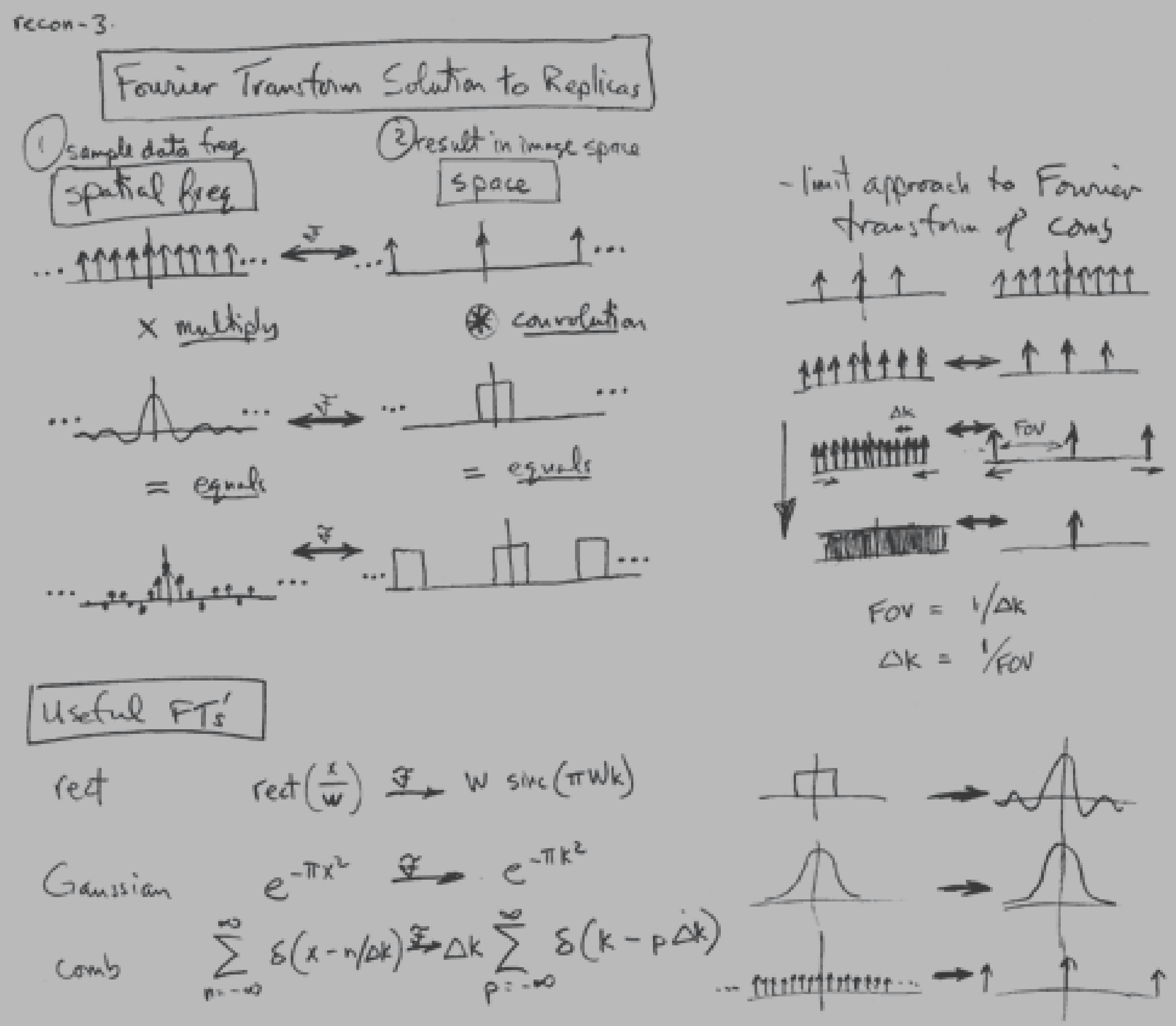
SAMPLING

aliasing, FOV

– must consider effects sampling limited points in k-space
[limited in range of frequencies sampled ($k_{min} \to k_{max}$)
limited in rate of sampling (Δk)

– N.B. aliasing less familiar when result of limited frequency domain sampling than limited space or time domain sampling




recon-3.

Fourier Transform Solution to Replicas

1 sample data freq

spatial freq

2 result in image space

space

× multiply

⊛ convolution

= equals

= equals

- limit approach to Fourier transform of comb

$FOV = 1/\Delta k$

$\Delta k = 1/FOV$

Useful FT's

rect $\quad \mathrm{rect}\left(\frac{x}{W}\right) \xrightarrow{\mathcal{F}} W\,\mathrm{sinc}(\pi W k)$

Gaussian $\quad e^{-\pi x^2} \xrightarrow{\mathcal{F}} e^{-\pi k^2}$

comb $\quad \sum_{n=-\infty}^{\infty} \delta(x - n/\Delta k) \xrightarrow{\mathcal{F}} \Delta k \sum_{p=-\infty}^{\infty} \delta(k - p\Delta k)$

When attempting to recreate what a subject is seeing with the help of the brain activity info obtained with an MRI scanner, little effort has yet been made to recreate colours. The focus has been on shapes, textures and semantic content.

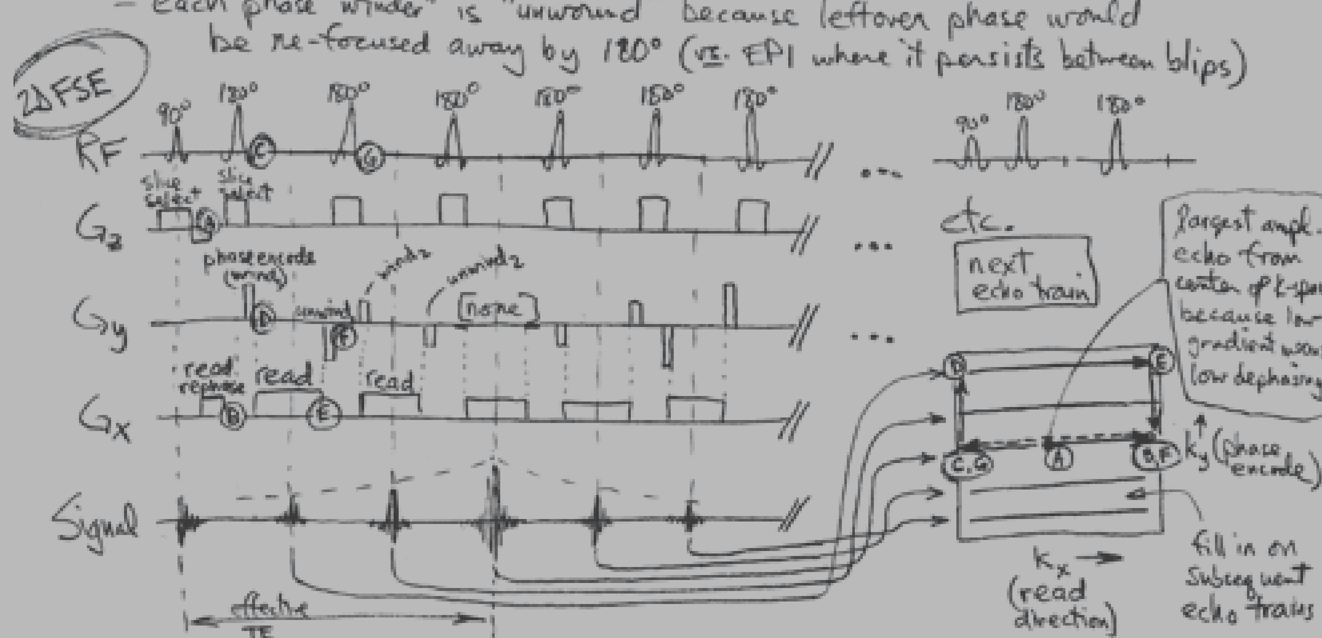
При попытках воссоздания того, что объект видит в результате информации о мозговой активности, полученной через МРТ, пока не было приложено особых усилий, чтобы воссоздать цвета. Основное внимание обычно отводится формам, текстурам и семантическому наполнению.

pulse-1

FAST SPIN ECHO (FSE)

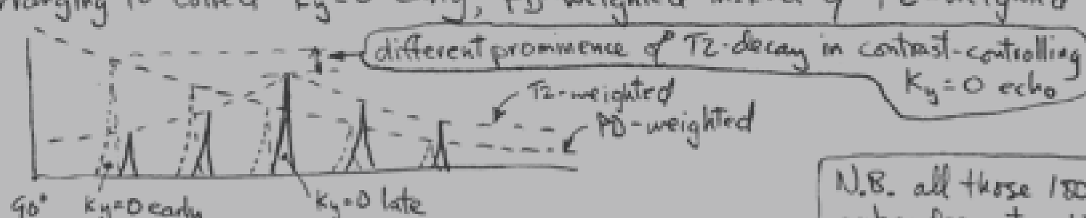
RARE, FSE, 3DFSE

- one 90° pulse followed by multiple 180° pulses (e.g., 8) "echo train"
  each with a different phase-encode gradient
- each phase "winder" is "unwound" because leftover phase would be re-focused away by 180° (vs. EPI where it persists between blips)



N.B.: only one read rephase; subsequent 180's reset from right to left

- the "effective TE" is the TE when center of k-space is collected (largest effect on contrast, longest echo)
- each subsequent echo has more T2 decay: $E_n = e^{-nTE/T2}$ $n = 1, 2, \ldots M$
- by arranging to collect $k_y = 0$ early, PD-weighted instead of T2-weighted



N.B. all those 180° pulses deposit a lot of RF power: $90° + 180° = \underline{45\times}$ power 30°

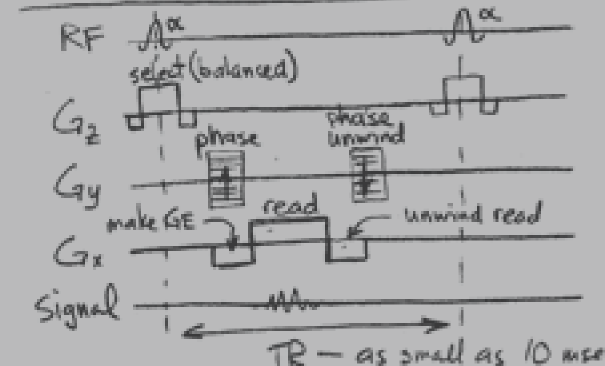
- possible to correct different T2-weighting of echoes by estimating T2 curve from $G_y=0$ echo train
- 3DFSE — like 2D except wind/unwind added to thick slice select (w/ crushers on 180°)

pulse-2

FAST GRADIENT ECHO

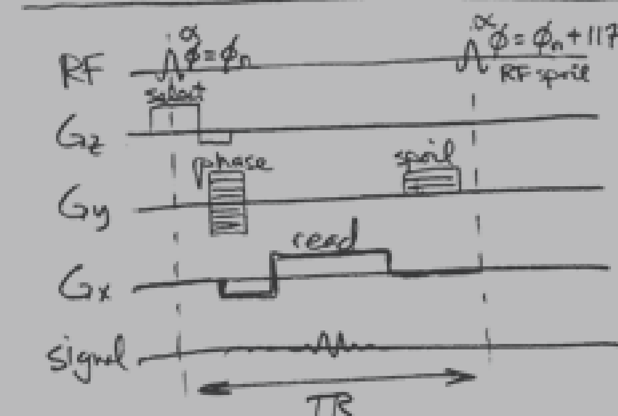
(GRASS, FLASH, FISP, SPGR, MPRAGE)

- small tip so TR can be greatly reduced (e.g. 10 msec, less than T2)
- 'leftover' undecayed transverse magnetization ["unwound" and re-used / "spoiled" before next shot



STEADY-STATE COHERENT (GRASS, FISP)

- unwind phase from phase-encode M_T before next pulse (there because TR < TE)
- unwind read gradient, too
- $\frac{T2}{T1}$-weighted (CSF very bright)



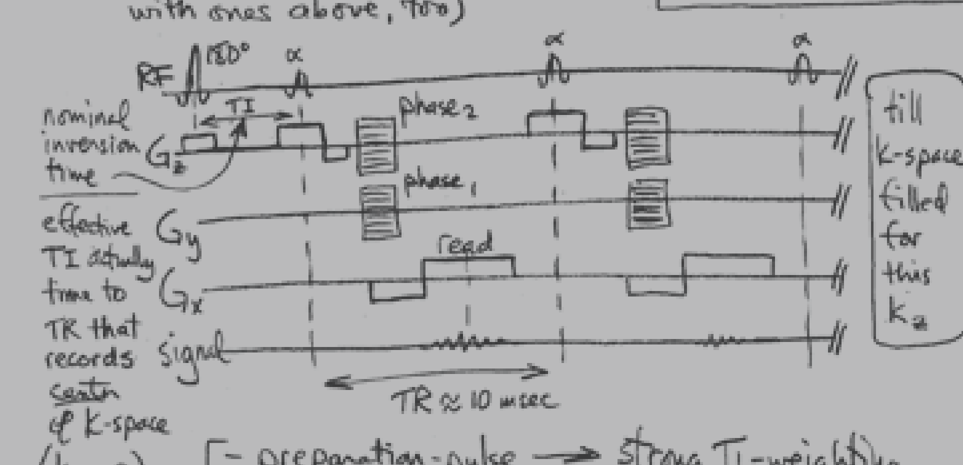
STEADY-STATE, SPOILED (SPGR, FLASH)

- spoil with random gradient (but this still allows some α refocusing)
- spoil with gradient plus incremented phase of RF pulses (RF spoiling)
- good gray-white contrast (T1-weighted)

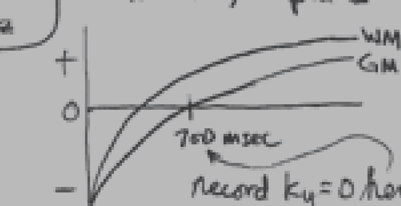
(shown as 3D sequence — possible with ones above, too)

NON-STEADY STATE, MAGNETIZATION-PREP.

(MP-RAGE)



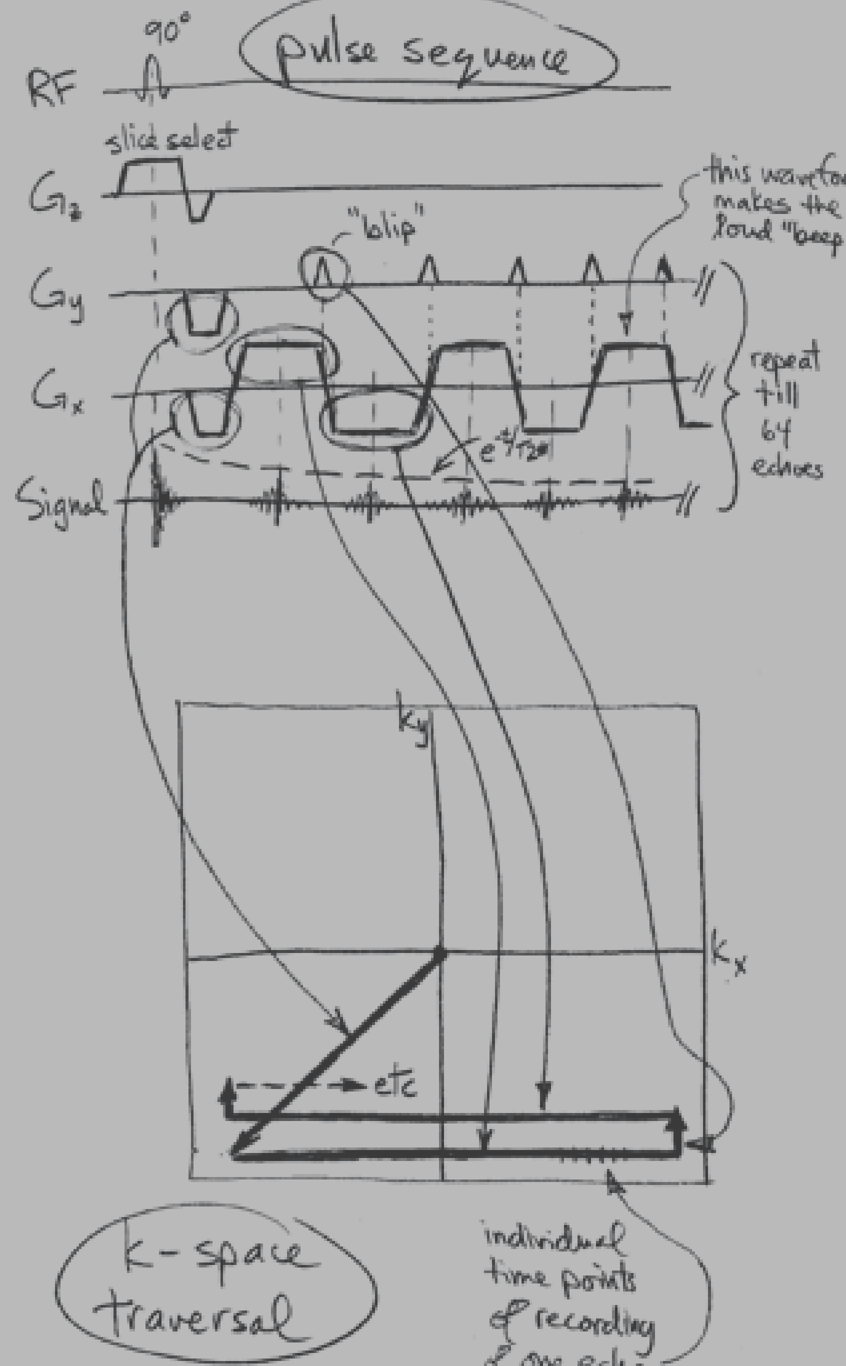
- longitudinal mag. not affect much by low angle pulses



[- preparation-pulse ⟶ strong T1-weighting
[- contrast varies in spatial-freq-dependent way

ECHO PLANAR IMAGING EPI (another fast gradient echo)

- single shot EPI collects all k-space lines (e.g. 64) after a 90° RF pulse using a train of gradient echoes



- since there is only one RF pulse per slice, spins never get reset to all-the-same (= zero freq, center of k-space)
- therefore, the recording point (Δt) in k-space (= spin phase stripe pattern) stays wherever the x and y gradients last left it
- that explains why successive y phase-encode steps are achieved without changing the size of the G_y "blips"
- echoes are $T2^*$-weighted (gradient echo)
- contrast mainly determined by echoes near center of k-space, which are only recorded after about 32 echoes

SPIN ECHO EPI why SE-BOLD may be selective for capillary bed

- standard EPI is a gradient echo method, which results in $T2^*$-weighting
- deoxyhemoglobin is paramagnetic, which reduces signal in a $T2^*$-weighted image due to greater dephasing
- the excess of oxyhemoglobin (probably the result of the need to drive O_2 into tissue, which requires more O_2 in the blood than is actually used) leads to the positive BOLD effect
- spin echo corrects (cancels) static $T2^*$ ($T2'$) dephasing, incl. deoxy
- if all spins stayed in the same position, spin echo would eliminate the BOLD effect by eliminating dephasing
- diffusion exposes spins to different fields (reducing gradient echo dephasing!)
- magnetic field gradients produced by large vessels are smoother across space than those produced by small vessels

- for TE ≈ 100 ms, spins diffuse 10's of μm, which is larger than diameter of small capillary, meaning that spins will likely experience different fields over time

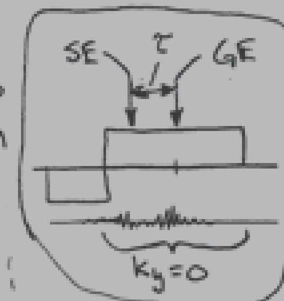
N.B.: this argument applies to the extravascular signal

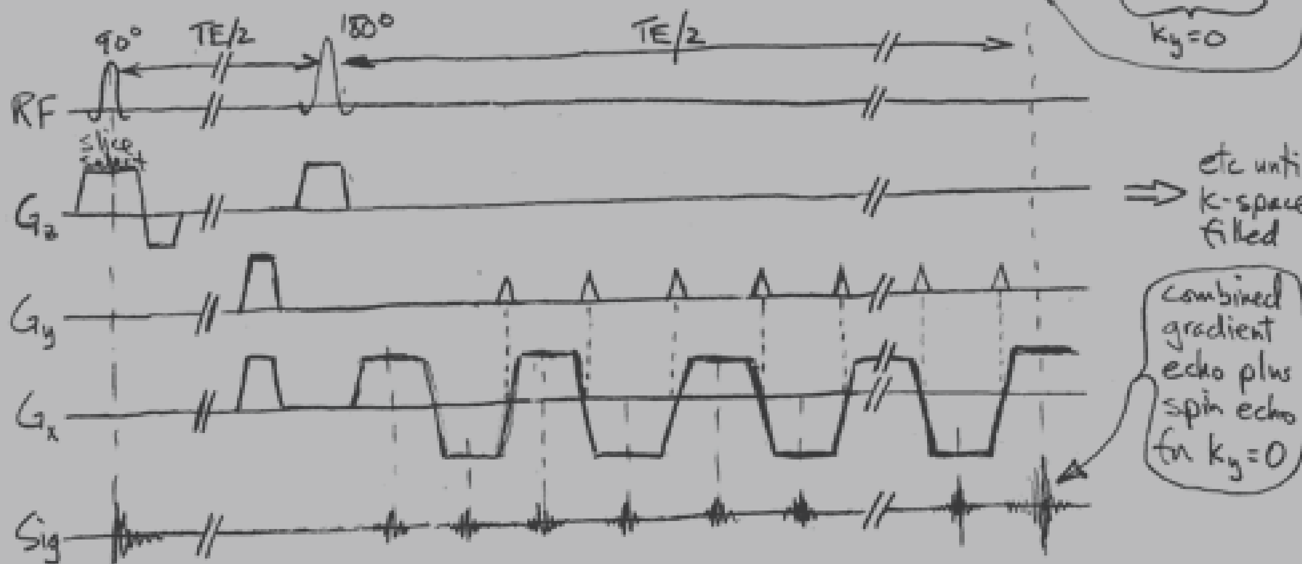
- therefore, spin echo will be less successful at canceling BOLD effect near small vessels (BOLD effect will be reduced near large vessels where diffusion less likely to expose spin to different fields here)

- this argument only works for extravascular spins — intravascular signal in BOLD is large (despite being only 4% by volume) because large gradient produced around red blood cells
  - → measure intra/extra w/ bipolar pulse which kills signal in faster moving blood in moderate and larger vessels ⇒ over half of SE-BOLD at 1.5T is venous...

SPIN ECHO EPI

- EPI is a multi-gradient echo pulse sequence
- "spin-echo EPI" uses a 180° pulse to add a single spin echo to the contrast-controlling gradient echo through the center of k-space
- "asymmetric spin-echo EPI" arranges for the spin echo to occur τ msec before the gradient echo, which gives more $T2^*$-weighting (for $k_y=0$ echo)





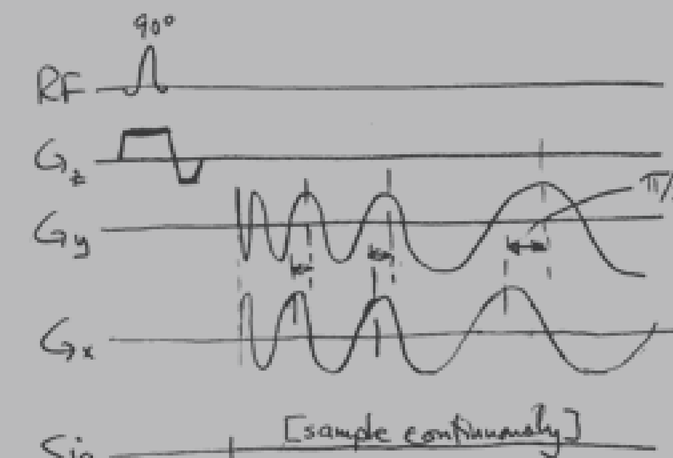
- the 180°-pulse rephasing reduces the $T2^*$ signal, which is why the partially rephased asymmetric spin echo has been more commonly used
- at higher fields, spin echo EPI is more promising
  - signal to noise is higher so we can take spin echo hit
  - contribution from venous blood is reduced, since blood T2 is so short, we can let it decay away before recording

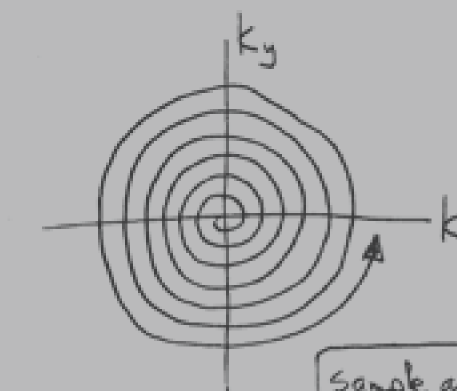
SPIRAL IMAGING

- by using smoothly changing gradients (sinusoids) less gradient power required than w/trapezoids (less eddy currents)

  earlier EPI hardware like this: sinusoidal gradient waveform from resonant circuit w/non-uniform sampling to get constant Δk_x

- sinusoids in both G_x and G_y give spiral k-space trajectory





Sample all orientations of each spatial frequency while slowly increasing spatial freq.

- constant angular velocity goes too fast at large k_x, k_y
- constant linear velocity better but impractical near $k_x=0$, $k_y=0$
- compromise: start constant angular, end constant linear → 

Constant angular velocity

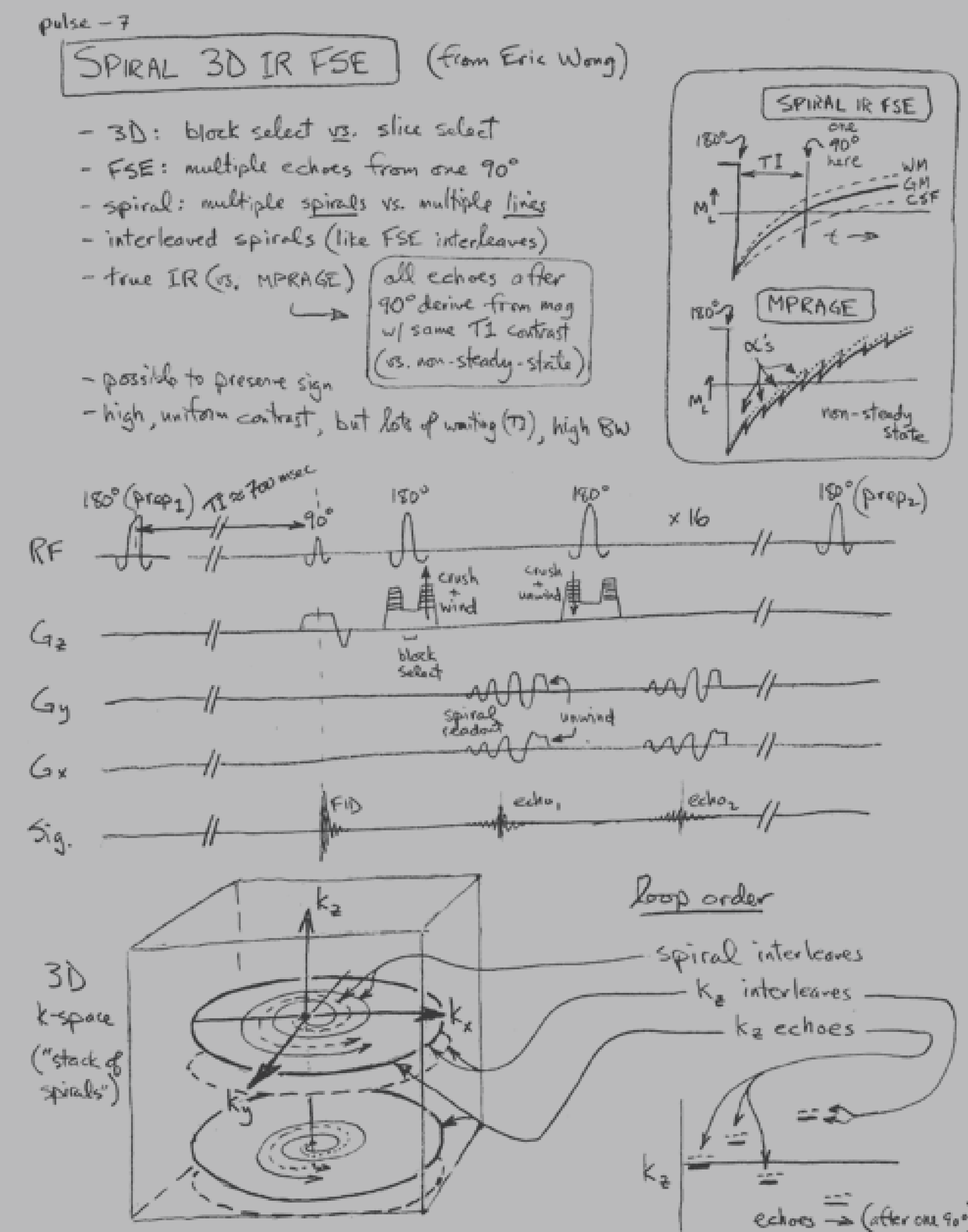
$$\omega(t) = \omega_0 t$$
$$\mathbf{k}(t) = A t e^{i\omega_0 t}$$
$$\mathbf{G}(t) = \frac{1}{\gamma}\frac{d}{dt}\mathbf{k}(t)$$
$$= A e^{i\omega_0 t} + iAt\omega_0 e^{i\omega_0 t}$$
$$G_x(t) = A\cos\omega_0 t - At\omega_0 \sin\omega_0 t$$
$$G_y(t) = A\sin\omega_0 t + At\omega_0\cos\omega_0 t$$

constant linear velocity

$$\omega(t) = \omega_0\sqrt{t}$$
$$\mathbf{k}(t) = A\sqrt{t}\, e^{i\omega_0\sqrt{t}}$$
$$\mathbf{G}(t) = \frac{1}{\gamma}\frac{d}{dt}\mathbf{k}(t)$$
$$= \frac{A}{2\sqrt{t}} e^{i\omega_0\sqrt{t}} + \frac{A}{2}\omega_0 e^{i\omega_0\sqrt{t}}$$
$$G_x(t) = \frac{A}{2\sqrt{t}}\cos\omega_0\sqrt{t} + \frac{A}{2}\omega_0\cos\omega_0\sqrt{t}$$
$$G_y(t) = \frac{A}{2\sqrt{t}}\sin\omega_0\sqrt{t} + \frac{A}{2}\omega_0\sin\omega_0\sqrt{t}$$

In laboratory conditions human subjects are able to discriminate between 700 and 900 simultaneous shades of grey.

В лабораторных условиях человеческие субъекты способны различать от 700 до 900 оттенков серого одновременно.

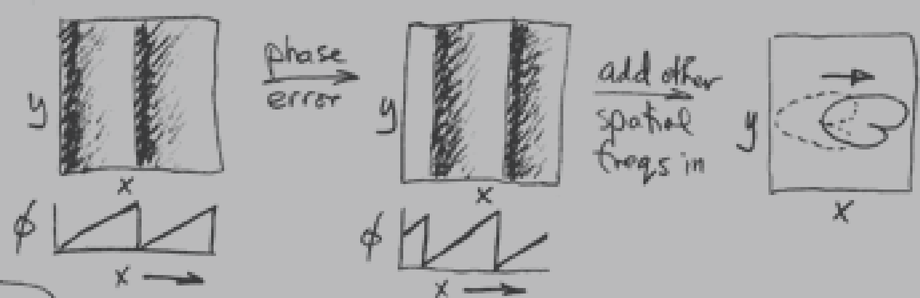


PHASE ERRORS & ECHO-CENTERING ERRORS

Phase Errors

- anything that causes a deviation of the B_z field strength from the expected value $(B_{0,z} + G_{x,z}x + G_{y,z}y + G_{z,z}z)$ changes precession frequency and therefore, expected phase angle
- incorrect phase of spatial frequency stripes results in a shift in space in the magnitude image after reconstruction

phase stripes in image domain (one k-space point)



Fourier shift theorem

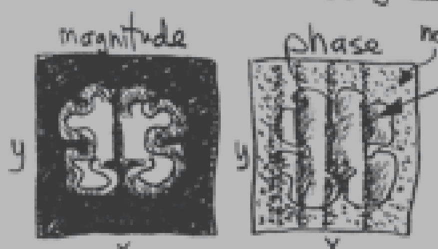
phase shift in spatial freq. domain causes spatial shift in image domain

$$\underbrace{I(x - x_0)}_{\text{x-offset in image}} = \int_{k_x} \underbrace{e^{-i2\pi k_x x_0}}_{\text{phase shift in signal}} \underbrace{S(k_x)}_{\text{k-space signal}} e^{i2\pi k_x x} dk_x$$

- correct w/ shimming and B_0-mapping/phase unwrapping before reconstruction

Echo Centering Error

- if realignment of all spins ($k_x = k_y = 0$) doesn't occur at the middle of read gradient, echo is shifted
- since echo is in spatial frequency domain, this is frequency shift
- spatial frequency shift results in wrapping in phase image after reconstruction
  → magnitude image unchanged



Fourier freq. shift theorem

freq shift in freq domain causes phase shift in spatial

$$\underbrace{e^{i2\pi k_{x_0} x}}_{\text{phase shift in reconstructed image}} \underbrace{I(x)}_{\text{image (complex)}} = \int_{k_x} \underbrace{S(k_x - k_{x_0})}_{\text{offset in spatial freq. space}} e^{i2\pi k_x x} dk_x$$

FAST SCAN ARTIFACTS EPI vs. Spiral

brain-induced field defects lead to phase errors

EPI

- G_x readout gradient strong → [field defects smaller percentage; less deformation of k_x (vertical stripe components)
- G_y "blips" are weak and total G_y record time much longer (5 times) than standard readout (50 ms vs. 10 ms)
- an extra gradient in the x-direction, for example, wraps and unwraps phase as a function of x-position
- but G_x big, so effect on freq.-encode direction is much less than on phase-encode direction, where error accumulates
- for a given x-position, the strength of the spurious gradient is constant, so the accumulation of phase error results shift in the y-direction (= k-space spin-stripe displ.)

- the phase error causes a shift in the y-direction proportional to x-gradient strength (= shear) but no blurring (N.B. shift varies w/x-position)

the lack of blurring has lead to a preference for EPI, despite the substantial image shifts

Spiral

- with center-out spirals phase errors accumulate in a radial direction

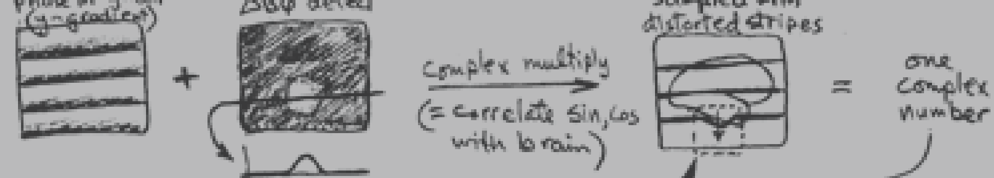
- thus, higher spatial frequencies have more error (= more shearing)
- for spurious x-direction gradient as above, there is a radial blurring, rather than a vertical shift because higher frequency phase stripes misaligned relative to low spatial freq

- for defects with more complex contours in the y-direction (than linear, as above) the vertical shifts (for EPI) will vary with y-position, and may result in signals from different y-positions being reconstructed on top of each other

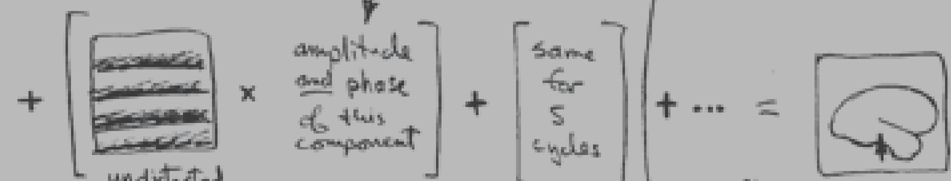
IMAGE-SPACE VIEW OF LOCALIZED $B\emptyset$ DEFECT, EFFECT ON RECON

- localized $B\emptyset$ defects often arise from air pockets embedded in tissue
  - air in middle/outer ear → indentation in inferior temporal lobe
  - air under olfactory epithelium → orbitofrontal ctx, ant. thal. compression

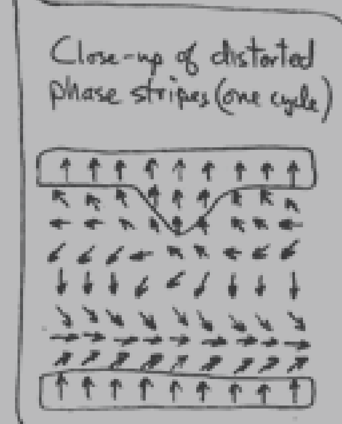
collect one data (k-space) point



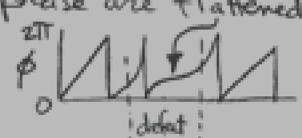
reconstruction from distorted data points



- same defect makes leftward dent in vertical phase stripes



- spatial information is lost when continuous changes in phase are flattened by $B\emptyset$ defect



- shifts can pile many pixels on top of eachother into one bright pixel

- local estimates of $\Delta B\emptyset$ needed to correct images
  1) fieldmap method:
     - multiple TE's to est. local $\Delta B\emptyset$ from ϕ/TE slope
     - shift each image pixel proportional to $\Delta B\emptyset$ in phase-encode dir.
  2) point-spread-function:
     - extra phase encode to estimate P.S.F. (should be δ-function)
     - deconvolve distorted image in phase-encode direction

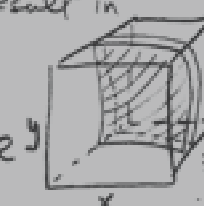
GRADIENT NON-LINEARITIES

< B_z
< B_x, B_y

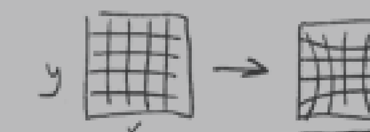
- ideally the G_x, G_y, and G_z gradient coils attempt to impress a linear variation onto to z-component of the B field — B_z — in the x, y, and z-directions

> these effects do not build up over time in phase-encode direction since they only occur when gradients are turned on

- in practice, gradient coils are non-linear (esp. printed-circuit-like)
- non-linearities are worse in smaller coils, but also in higher performance coils, designed for post-processing correction of distortions
- non-linearities result in phase errors, which result in 3-D image distortion
  - a non-linear slice-select gradient will excite a curved slice



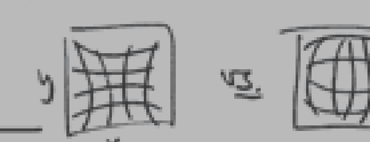
  - non linear phase and frequency encode gradients will distort in-plane features



- some scanners correct these differently for 3-D scans (all directions), 2-D scans (just inplane), and EPI scans (no corrections!)

> these distortions are predictable and can be corrected

- this can result in errors approaching 1 cm in funct-struct overlays
- different coil designs have different directions of distortion (!)



- the assumption of non-Maxwellian gradients results in additional phase errors

> that is, the assumption that gradients cause no field in the B_x and B_y direction

- these can also be corrected since the B_x and B_y components are known

SHIMMING AND B_0-MAPPING

- passive iron shims inserted to flatten B_0 field
- additional coils (usu. in the gradient coils) can be statically energized in an attempt to flatten the B_0 field (a few ppm good)
- primary use is to compensate for defects in flatness present without a sample in the magnet (geometric imperfections, impurities in metal, etc) ($\approx$ several hundred ppm)

[linear shim coils impose gradients in x, y, and z
higher order shims impose higher order spherical harmonic field components (e.g. z^2)

- secondary use is to compensate for inhomogeneities caused by introducing the sample into the B_0 field
- local resonance offsets caused by B_0 defects estimated from images
  - ↳ e.g., sample phase at multiple echo times

(after correcting wraps)
slope is resonance offset
⇓
because: longer time to echo means more time for phase to be retarded (or advanced) by B_0 defects

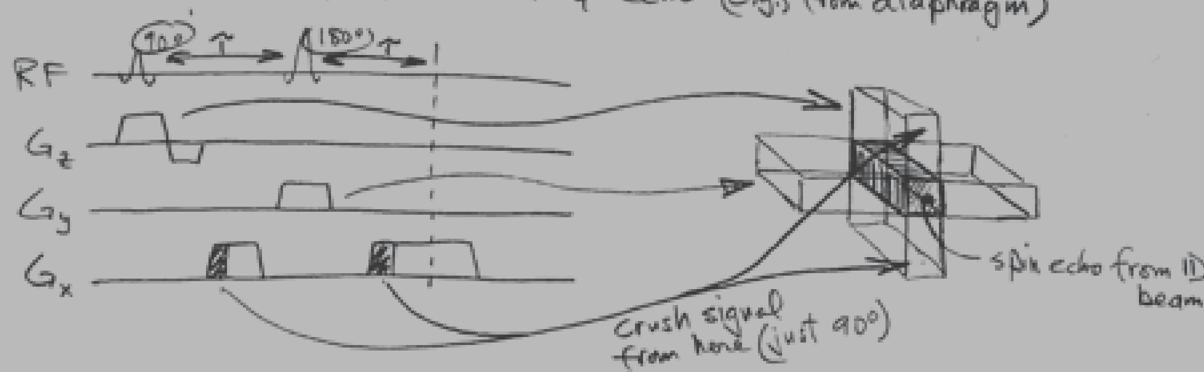
- fit defective field using combination of fields generated by shim coils. then <u>add</u> these corrections to base shim currents
  - ↳ this only corrects <u>spatially</u> <u>gradual</u> field defects
  - ↳ local defects due to air in sinuses much higher order than shims
- after shimming, field map measured again
- image voxel displacements calculated from resonance offset map are used to unwarp the reconstructed magnitude image
- for EPI images, assume displacements all in phase-encode direction (since freq encode gradient is strong relative to defects)

Experiments have been made to recreate people's dreams with MRI technology. The biggest problem has not been to record the brain activity of the sleeping subject, but to confirm the results. Memories of dreams are notoriously elusive.

Были проведены эксперименты по воссозданию снов человека с помощью технологии МРТ. Самой большой сложностью, однако, стала не сама запись мозговой активности спящего человека, а подтверждение результатов. Память о снах известна своей ускользаемостью.

NAVIGATOR ECHOS for motion correction

- 1D navigator: excite beam of spins, detect movement from k-space shift of echo (e.g., from diaphragm)

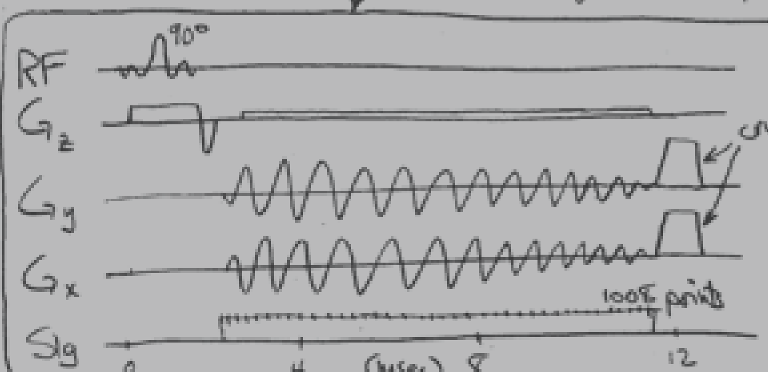


- 3D navigator: collect 3D sphere in k-space

[rotation of object → rotation of k-space amplitude pattern
translation of object → phase shift of k-space phase (Fourier shift)

- sample at sufficient k-space radius to pick up high spatial freq features
- N.B.: excite whole volume
- do N,S hemispheres separately (less T2*, cancel EPI-like error accumulation)
  - ↳ equator → up, equator → down

Welch et al. (2002) MRM



$$z(n) = \frac{2n - N - 1}{N}$$ total pnts

$$y(n) = \cos\left(\sqrt{N\pi}\,\sin^{-1} z(n)\right)\sqrt{1-z^2(n)}$$

$$x(n) = \sin\left(\sqrt{N\pi}\,\sin^{-1} z(n)\right)\sqrt{1-z^2(n)}$$

(skip poles — slew rate too high)

- can be used for prospective motion correction (rotate, translate w/ gradients)
- better estimate, because of speed, than full TR of EPI images (27 ms vs. 2-4 sec)
- may need to smooth rot, trans estimates across time (e.g. Kalman filter)

RF FIELD INHOMOGENEITIES B_1 inhomogeneities

- receive coil inhomogeneities alter the amplitude of the received signal, altering the reconstructed proton density in a spatially varying way
  - ↳ variations can be used (cf. SMASH, SENSE) or corrected

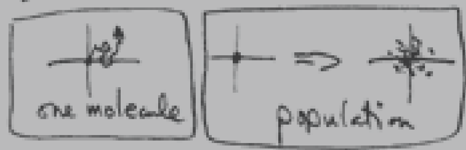
- transmit coil inhomogeneities affect the flip angle in a spatially varying way
  - ↳ potentially worse (why local transmit is rare)
  - ↳ usu. fixed by using a large transmit coil (e.g. body coil)
- RF penetration at higher fields (= higher RF frequencies) is less uniform:
  1) decreased RF wavelength (closer to size of head) at higher freq.
  2) increased permittivity (ε) and conductivity (σ) at higher field
- the advantage of the falloff in signal seen with a small, receive-only RF coil is better signal-to-noise (less noise received from other parts of brain)
- different sensitivity functions from different coils can be used to scan less lines in k-space

SMASH — use weighted sensitivity profiles to form sinusoidal harmonics which are then used to fill in missing k-space lines

SENSE — subsample k-space (like SMASH) then unfold aliased images in image domain after estimating sensitivity profile

SPACE RIP — simple linear inverse after 1D FT (more general) diffusion-1

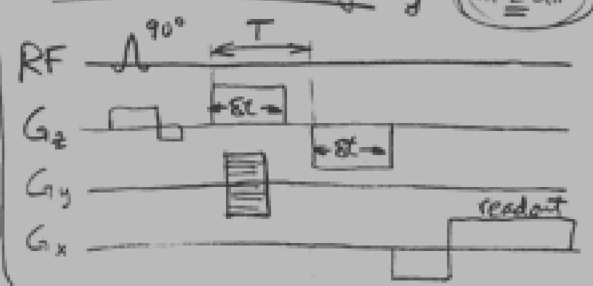
DIFFUSION - WEIGHTED IMAGING



$\sigma_x = \sqrt{2DT}$ — classical diffusion coefficient; time

e.g. brain $D = 0.001\ mm^2/sec$

Simple diffusion weighting (in z dir)



- spins acquire phase during first δt
- if spins diffuse (= move) along gradient by time T, signal is lost because negative δt doesn't re-phase
- attenuation: $A(D) = e^{-bD}$

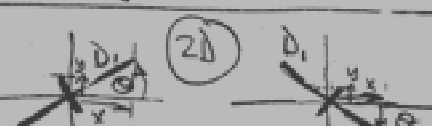
  where $b = (\gamma G \delta t)^2 (T - \delta t/3)$

- "apparent diffusion coefficient" map → calculate D image from $b=0$, $b=$ large
- to get large b, need $G\uparrow$, $\delta t\uparrow$ (need big G's)
  - long δt gives spurious T_2-weighting
  - use stimulated echos: 90°RF — δt_1 — 90°RF — 90°RF — δt_2 (allows long T)

    90°RF ⇓ transverse; δt_1 diffusion lobe 1; 90°RF ⇓ park in longitudinal; 90°RF ⇓ back to transverse; δt_2 diffusion lobe 2

Anisotropic Diffusion (Gaussian)

- measure D along multiple axes
- have to measure tensor, not scalar →
  - even for determining one primary direction

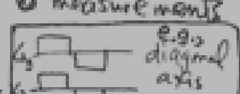


- without 3rd number (θ) x & y projections same

$$D = [u_x\ u_y\ u_z] \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

- since **D** is symmetric, only need 6 measurements → i.e., paired diffusion lobes applied along 6 axes

$$\underbrace{D}_{\text{scalar diffusion}} = \mathbf{u}^T \times \underbrace{\mathbf{D}}_{\text{diffusion tensor}} \times \underbrace{\mathbf{u}}_{\text{measurement direction}}$$



Diffusion Surface (non-Gaussian)

- need to measure diffusion in many directions
- e.g., icosahedral subtesselation (126)
- required for even two fiber directions

Fiber tract mapping

- an ill-posed problem
- e.g., constrain both ends and center point (!!)
- need vis. areas test

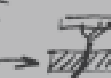
perfusion-1

PERFUSION - ARTERIAL SPIN LABEL

(like inversion recovery)

- basic idea:
  - tag blood below area of interest
  - collect control & tagged image
  - assume directional input flow!
  - tag is 180° pulse
  - sign not problem when delay long enough (see below)
- continuous ASL (CASL) —
  - continuously tag a plane →

  - gradient on, blood gets adiabatically inverted as it passes through location w/ correct resonant freq.
- pulsed ASL (PASL) — e.g., EPISTAR, FAIR, PICORE, QUIPPS II
  - tag block of tissue below slice(s) →



- small diffs between control and tag (~1%)
  - requires accurate balancing of control & tag images
- contrast problems:
  - transit delays → biggest confounding factor
  - relaxation rate diffs
  - venous clearance (vs. microspheres, which get stuck!)
  - transit delays:

TI →	500	1000	1500 msec
	large vessels	frontal temporal parietal	occipital

solutions for quantitative
- insert delay so all spins arrive into low velocity capillary
- kill end of tag to reduce spatial variation of tag

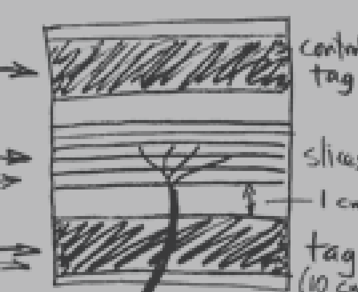
QUIPSS II — quantitative perfusion

TURBO ASL
- use TI longer than TR
- omit QUIPSS tag ending

tag pulse
control img.
control pulse
tag image
tag pulse
control img.

2x faster but limited slice #

1) pre-saturate spins in target slices
2) tag - 180° pulse below slices; control - 180° pulse above slices (to control off-resonance)
3) saturate tagged block to end tag (TI_1)
   - both tag and control
   - can use train of thin slices pulses at top of tag band
4) EPI or spiral images of target slices (TI_2)
   - image most distal slice last to cancel delays
   - fast between slice so imaging excitations don't get interpreted as flow

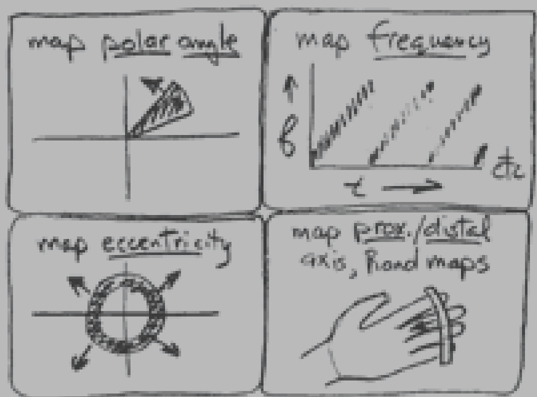


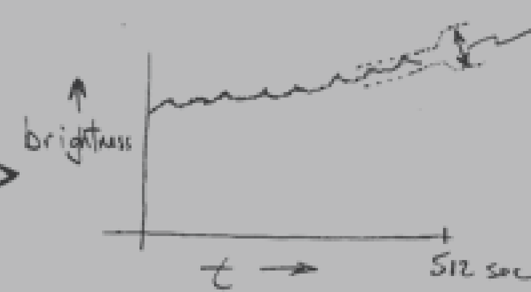
$$\Delta M \cong \text{flow} \times \left[2M_0\ TI_1\ e^{-TI_2/T1A} \right]$$

can extract flow and BOLD — adjacent subtractions minimize movement artifact

1) alternate tag and control, GRE TE=30 ms
   - both BOLD-weighted
   - control - tag → flow
   - control + tag → BOLD
   - tag - control - tag - control ...
2) dual echo spiral
   - k=0 early ⇒ hi S/N flow
   - TE = 30 ms ⇒ BOLD

PHASE-ENCODED STIMULUS & ANALYSIS

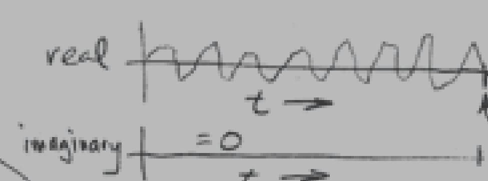




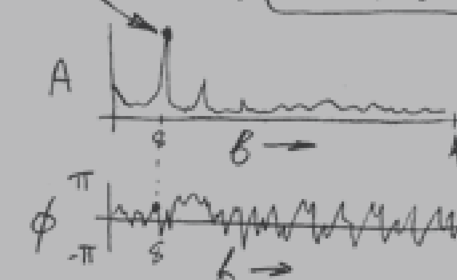
periodic stimuli (phase-encoded) — e.g., 8 cycles at 64 sec/cycle

strongly periodically activated single voxel time course

remove constant (avg) and linear trend



FFT, convert to A, ϕ



calculate significance

- ratio between amplitude at stimulus frequency (= signal) and average of amplitudes at other frequencies (= noise)
- ignore harmonics, lo freq (= movement)

smooth

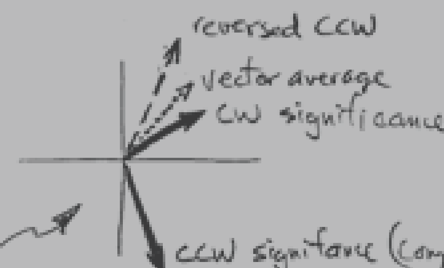
- vector average of complex significance (A, ϕ) with that at nearest neighbor surface points

display

- plot phase using hue and saturation to indicate significance

delay correction

- record responses to opposite directions of stimulus (ccw/cw, in/out, up/down)
- vector average after reversing angle of one
  - penalizes inconsistent more than just avg of angles



CONVOLUTION

for one x

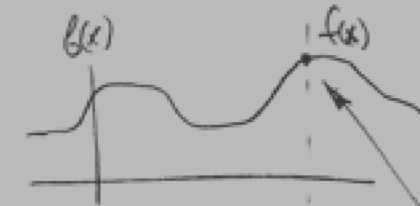
$$f(x) = g(x) * h(x) = \int_{z=-\infty}^{z=+\infty} \underbrace{g}(z) \cdot \underbrace{h}(x-z)\, dz$$

– definition of convolution

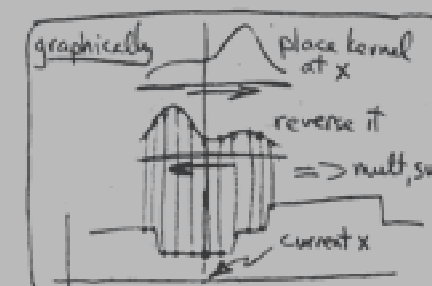
sum this product across all $\underline{z}$

kernel

"image"

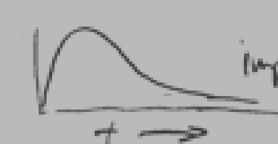


- how to calculate one term
- sum across all $\underline{z}$ to get the value of the convolution at at $\underline{x}$
- move kernel to calculate next $\underline{x}$

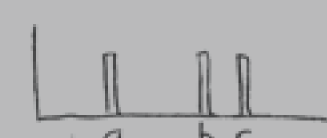


why we reverse

N.B. cross-corr same as convolution except no reversal
h(x+z) instead of h(x-z)



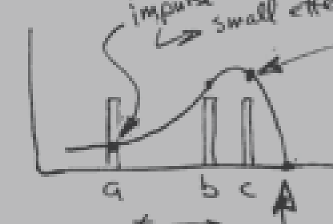
impulse response function (HRF)



impulses (erp design)

impulse occurred a while ago
↳ small effect

impulse occurred recently
↳ large effect



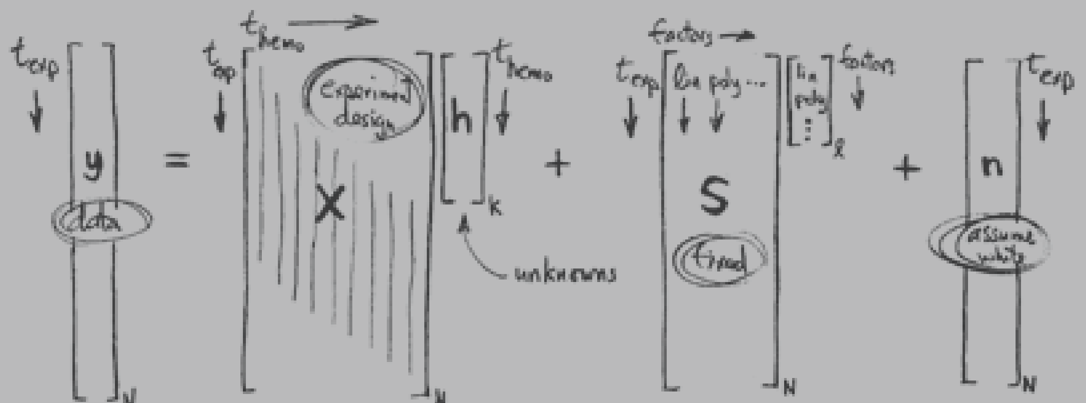
how to calculate convolution for this time point (only 3 terms in integral — all other zero)

General Linear Model

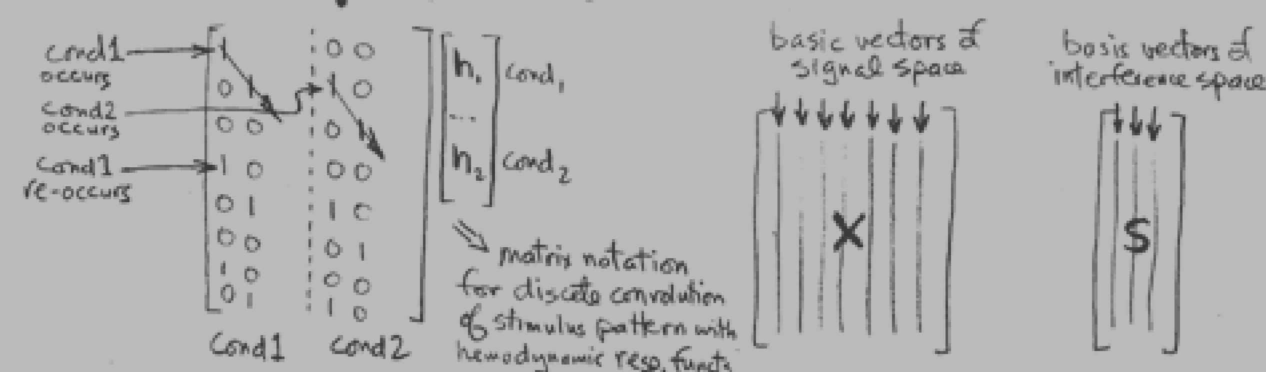
$$\vec{y} = \overset{\square}{X}\vec{h} + \overset{\square}{S}\vec{b} + \vec{n}$$

data = design·HDR + drifts·weights + noise

– goal is to solve for the hemodynamic response functions, $\mathbf{h}$



⇓ multiple conditions



maximum likelihood estimate

1) assume white noise, solve for $\vec{h}$

2) $\hat{h} = (X^T P_S^{\perp} X)^{-1} X^T P_S^{\perp} y$ where $P_S^{\perp} = I - S(S^T S)^{-1} S^T$

↳ projection matrix that removes part of vector that lies in $\mathbf{S}$ space

or

$= (X_\perp^T X_\perp)^{-1} X_\perp^T y$ where $X_\perp = P_S^{\perp} X$

↳ design matrix w/nuisance effects removed from col's

3) significance (how to construct F-ratio)

$$F = \frac{N-k-\ell}{k}\left[\frac{y^T(P_{XS}-P_S)y}{y^T(I-P_{XS})y}\right]$$

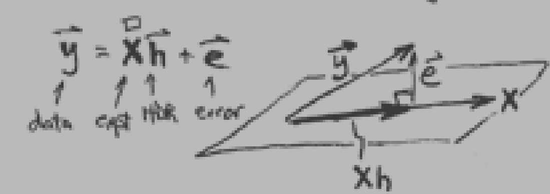
⟶ see diagram next page for geometric interp

Geometric Interpretation Of General Linear Model

– with no nuisance functions (S), we could look at orthogonal projection of data onto experimental design and compare that to error to determine significance

$$\vec{y} = \overset{\square}{X}\vec{h} + \vec{e}$$

data, expt, HDR, error



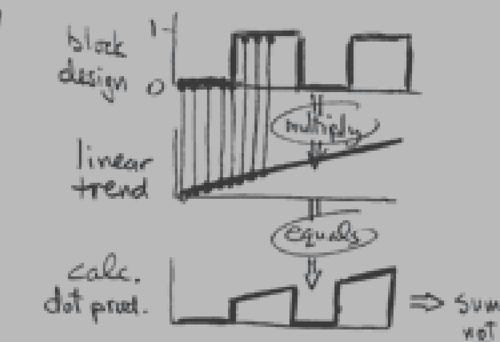
$\overset{\square}{X}\vec{h} = \overset{\square}{P}_x\vec{y}$

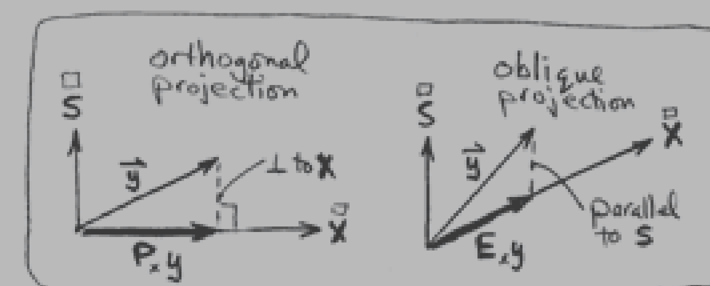
projection matrix, P_x, operates on $\vec{y}$ to give projection of data into experiment space, X

– when nuisance functions, $\overset{\square}{S}$, are considered, problem: $\overset{\square}{S}$ may not be orthogonal to $\overset{\square}{X}$

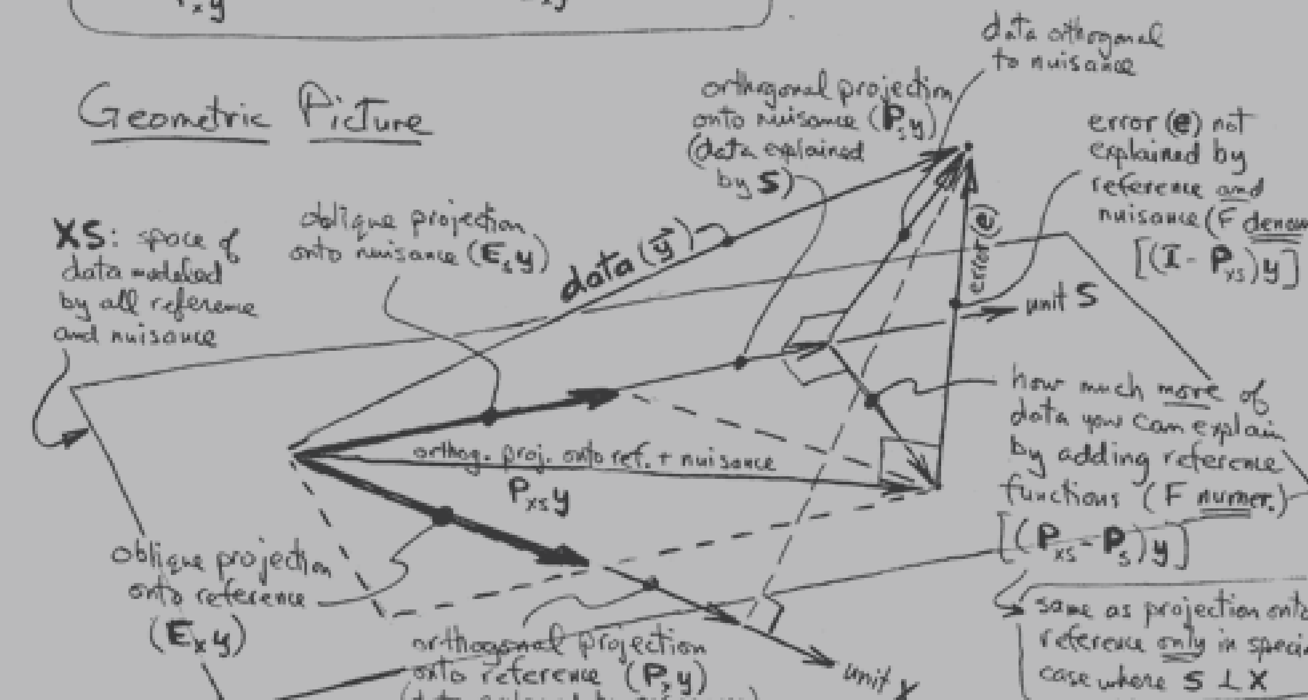
↳ for example: linear trend not orthogonal to std. block design

[remember: "orthogonal" means [dot prod. = 0; corr = 0]]





Geometric Picture



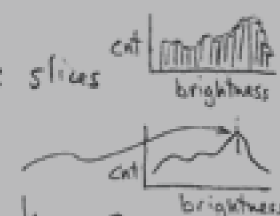
SEGMENTATION & SURFACE RECON
Talairach, Normalize, StripSkull

1) <u>MNI auto-Talairach</u> → generates 4×4 matrix

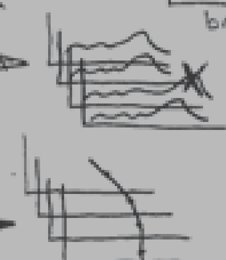
$$\left[\begin{array}{ccc|c} & \begin{matrix}3\times3\\ \text{(rotate}\\ \text{\& scale)}\end{matrix} & & \begin{matrix}x\\y\\z\end{matrix} \\ 0 & 0 & 0 & 1\end{array}\right]$$ translate

- make average brain target (blurry)
- blur target (further), blur single brain (a bit), gradient descent on xcorr
- repeat w/ less blurring of avg target and current brain
- problems: variable neck cutoff
  ↳ but <u>much</u> better than standard! – only 2 points near <u>center</u> of brain! – fit to bounding box

2) <u>Intensity Normalization</u> (output: "T1")



- histogram of pixel values in 10 mm thick HOR slices
- Smooth histogram
- peak find to get initial estimate of white matter
- discard outlier peaks across slices →
- fit splines to peaks across slices
  ↳ <u>interpolates</u> scaling factor ⊥ to HOR
- scale each pixel so WM peak is 110 →

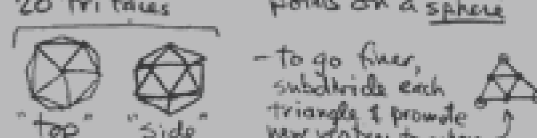


- refine estimate to interpolate in 3D

5-10 times:
- find points in 5×5×5 within 10% of WM, get new scale for them
- build Voronoi to interpolate scales <u>unset</u> above
- soap-bubble-smooth Voronoi boundaries (3 iterations)
- re-scale each voxel

3) <u>Skull Stripping</u> (output: "brain")

12 vertices → even distribution of points on a <u>sphere</u>
20 tri faces



- to go finer, subdivide each triangle & promote new vertices to sphere

- "shrink-wrap" algorithm
- start with ellipsoidal template → subtesselated icosahedron
- minimize <u>brain penetration</u> and <u>curvature</u>
- <u>curvature</u>: spring force (from center-to-neighbor vect sum)

- decompose into ⊥ and tangential (local normal from summed normed cross products)
- <u>brain penetration</u>
  apply force along surface normal that prevents surface from entering gray matter

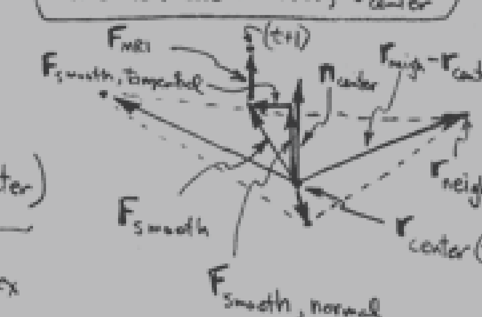
SEGMENTATION & SURFACE RECON
spring force in detail

- implementing a "force" is like directly constructing the operator that minimizes something (without first defining the "something")
- more formally, we would define cost function, <u>then</u> take its derivative (gradient) to minimize it

<u>Shrinkwrap update eq.</u> (skull strip, original Dale & Sereno surface refinement)

$$\underbrace{\mathbf{r}_{center}(t+1)}_{\text{for one vertex}} = \underbrace{\mathbf{r}_{center}(t)}_{\text{previous}} + \underbrace{\mathbf{F}_{smooth}(t) + \mathbf{F}_{MRI}(t)}_{\text{local quantities}}$$

rule for each vertex, $\vec{\mathbf{r}}_{center}$

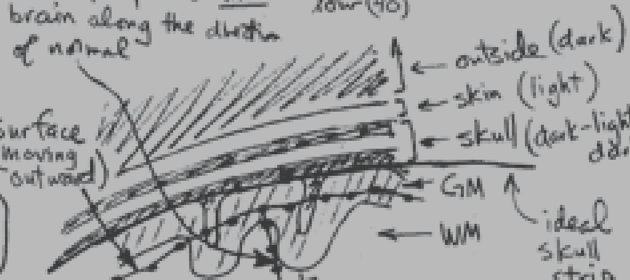


$$\mathbf{F}_{smooth} = \underbrace{\lambda_{tang}}_{\substack{\text{stronger}\\ \text{than normal}\\ (0.5)}} \sum_{neigh} \underbrace{(\overset{\text{identity } 3\times3}{\mathbf{I}} - \mathbf{n}_{center}\mathbf{n}^T_{center})}_{\substack{\text{expand by distribution to:}\\ \text{neighbor vector minus projection}\\ \text{of neighbor onto normal} \to \text{tangential!}}} \underbrace{\cdot}_{\text{project onto}} \underbrace{(\mathbf{r}_{neigh} - \mathbf{r}_{center})}_{\substack{\text{vector to}\\ \text{neighbor vertex}}}$$

$$+ \underbrace{\lambda_{normal}}_{\substack{\text{weaker}\\ \text{than tangential}\\ (0.1)}} \left[\sum_{neigh} \underbrace{(\mathbf{n}_{center}\mathbf{n}^T_{center}) \cdot (\mathbf{r}_{neigh} - \mathbf{r}_{center})}_{\substack{\text{projection of } \underline{a} \text{ neighbor vertex vector onto normal in the}\\ \text{direction of the normal (}\mathbf{n}\text{ is squared (as shown) so we get}\\ \text{a vector out (not a scalar)}}} \underbrace{-}_{\substack{\text{vector,}\\ \text{subtract off}}} \underbrace{\frac{1}{\#\text{vertices}} \sum_{v} \sum_{neigh} (\mathbf{n}_v\mathbf{n}^T_v) \cdot (\mathbf{r}_{neigh} - \mathbf{r}_v)}_{\text{average normal component}} \right]$$

$$\mathbf{F}_{MRI} = \underbrace{\lambda_{MRI}}_{\substack{\text{strong}\\ (1.0)}} \mathbf{n}_{center} \underbrace{\prod_{d}^{30}}_{\substack{\text{multiply all terms} \to\\ F_{MRI}=0 \text{ if } \underline{\text{any}} \text{ are zero}}} \max\Big[\underbrace{0,}_{\substack{\text{don't}\\ \text{allow neg}}} \underbrace{\tanh}_{\substack{\text{max force saturates at 1.0}\\ (\text{max "}\Pi\text{" product} = 1.0)}} \big[\overset{\text{intensity MRI data}}{I} \underbrace{(\mathbf{r}_{center} - d\,\mathbf{n}_{center})}_{\substack{d \text{ sample points } \underline{\text{into}}\\ \text{brain along the direction of normal}}} - \underbrace{I_{thresh}}_{\text{low (40)}}\big]\Big]$$

Snapshot of surface and "core sample" from one vertex



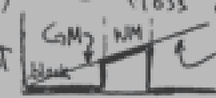
surf-3

SEGMENTATION & SURFACE RECON

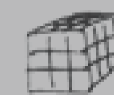
filter, cut, tessellate

4) Non-isotropic filtering (output: "wm") — "floss" and "speckle"

- preliminary hard threshold:



- find ambiguous/boundary voxels
  ↳ 20% or more of 26 immediate neighbors different



↳ to avoid expensive calc below...

- find plane of least variance

for each ambiguous voxel
- for each direction (from icosahedral supertessellation)
- consider 5x5x5 volume around 1 voxel
- find plane of least variance in this hemisphere (1 vox thick)
- median filter w/ hysteresis
  ↳ if 60% of within-slab differ, reverse classification
  ↳ "flosses" sulci without blurring

5) Find cutting planes

- callosum, to separate hemispheres (SAG)
- midbrain, to avoid fill into cerebellum (HOR)

→ Talairach to start; fill WM in SAG or HOR till min area

6) Region-growing to define connected parts (output: "filled")

- inside-out, outside-in, inside-out — for each hemisphere
- up/down cycles within each plane
- plane-by-plane

- "wormhole filter" (3x3x3 = center + 26)
  ↳ fill (unfilled) voxel if 66% neighbors differ → eliminates structures w/ thin, 1-D structure

In the mid 20th century it was commonly believed that we dream in black and white. This was probably due to the fact that a lot of everyday imagery was still predominantly black and white in those days, be it newspapers, photography or television. Dreams might be of unspecified colour, but there is nothing preventing us from dreaming in colour.

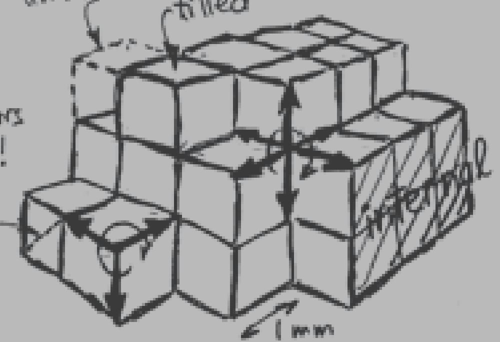
В середине 20 века существовало расхожее мнение, что нам снятся только черно-белые сны. Возможно, это можно объяснить тем фактом, что в большинстве своем совокупность изображений в то время были черно-белыми, будь то газеты, фотографии и или телевидение. Сны могут быть неопределенного цвета, но ничто не мешает нам видеть сны в цвете.

surf-4

SEGMENTATION & SURFACE RECON   3D → 2D

7) Surface Tessellation (output: rh.orig, lh.orig)

- variable num neighbors possible!
- quads to triangles



3D data set → $I(x,y,z)$

⇓

2D surface → vertex list (coords)

$x_1\ y_1\ z_1$
$x_2\ y_2\ z_2$ ← vtx nums implicit
⋮

→ face list (vertex nums)

$a_1\ b_1\ c_1\ d_1$
$a_2\ b_2\ c_2\ d_2$ ← face nums implicit
⋮

- find filled voxels bordering unfilled
- make ordered list of neighboring vertices
  ↳ so cross-products oriented properly
- long list of values associated with each numbered vertex
  e.g.
  - position (orig, morphed)
  - area (orig, morphed)
  - curvature (intrinsic, Gaussian)
  - "sulcusness" (summed ⊥ movement during unfolding)
  - cortical thickness
  - fMRI data ⇐
  - EEG/MEG dipole strength
- separate fMRI data set must be aligned, sampled
  - fMRI voxels larger
  - sample at each surface vertex
  - nearest-neighbor "soap bubble" smoothing to interpolate data onto hi-res mesh
- some quantities only well-defined on surface
  ↳ gradient of magnitude of cortical map measure (e.g., eccentricity)

fMRI → surface initial sample same value here

⇓

interpolate on surface

surf-5

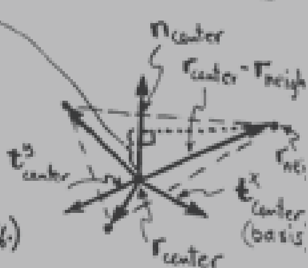
SEGMENTATION & SURFACE RECON   smooth, inflate, final surfaces

- smoothing/inflation/WM, pial done as derivative of energy functional

$$\underbrace{J}_{\text{total scalar error to minimize}} = \underbrace{J_{tangential}}_{\text{scalar tangential error (fixed by redistributing vertices)}} + \underbrace{\lambda_{normal}}_{\text{small (0.25)}} \underbrace{J_{normal}}_{\text{scalar curvature error (fixed by reducing curvature)}} + \underbrace{\lambda_{image}}_{\text{smaller (0.075)}} \underbrace{J_{image}}_{\text{scalar image error (fixed by moving toward target image value)}}$$

$$\underbrace{J_{normal}}_{\text{across all vertices}} = \underbrace{\frac{1}{2\,\#vert}}_{\text{1/2 so no coefficient on derivative}} \underbrace{\sum_{centers}}_{\text{across all vertices}} \underbrace{\sum_{neighbors}}_{\text{neighbors of one vertex}} \left[\underbrace{\mathbf{n}_{center}}_{\text{vertex unit normal}} \cdot \underbrace{(\mathbf{r}_{center} - \mathbf{r}_{neigh})}_{\text{vector from current center to one neighbor (position vector diff.)}}\right]^2$$

project onto normal



$$J_{tangential} = \frac{1}{2\,\#vert} \sum_{centers} \sum_{neighbors} \left[\underbrace{\mathbf{t}^x_{center}}_{\text{x-direction in tangent plane}} \cdot (\mathbf{r}_{center} - \mathbf{r}_{neigh})\right]^2 + \left[\underbrace{\mathbf{t}^y_{center}}_{\text{y-direction in tangent plane}} \cdot (\mathbf{r}_{center} - \mathbf{r}_{neigh})\right]^2$$

project vector to neighbor onto x & y

$$J_{image} = \frac{1}{2\,\#vert} \sum_{centers} \left[\underbrace{I^{targ}_{center}}_{\text{target brightness}} - \underbrace{I(\mathbf{r}_{center})}_{\text{brightness at current location}}\right]^2$$

I^{targ} for WM: mean of voxels labeled WM in 5 mm neighborhood

I^{targ} for pia: global — small num for C.S.F.-like

- take directional derivative of energy functional (to find steepest uphill)
- move each vertex in the opposite (negative) direction w/ self-intersect test

N.B.: eq. 9 in Dale, Fischl & Sereno different — and incorrect!

$$\underbrace{-}_{\text{go the opposite direction}} \underbrace{\frac{\partial J}{\partial \mathbf{r}_{center}}}_{\text{direction (vector) of largest scalar error for one vertex}} = \lambda_{image} \left[I^{targ}_{center} - I(\mathbf{r}_{center})\right] \underbrace{\nabla I(\mathbf{r}_{center})}_{\text{vector — calculate gradient on blurred-by-Gaussian image}}$$

$$+ \sum_{neighbors} \lambda_{normal} \left[\mathbf{n}_{center} \cdot (\mathbf{r}_{center} - \mathbf{r}_{neigh})\right] \underbrace{\mathbf{n}_{center}}_{\text{unit normal vector scaled by dot product}}$$

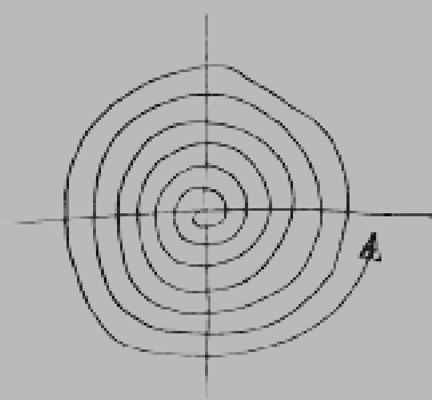
$$+ \sum_{neighbors} \overbrace{\left[\mathbf{t}^x_{center} \cdot (\mathbf{r}_{center} - \mathbf{r}_{neigh})\right] \mathbf{t}^x_{center}}^{\text{x-component of tangential}} + \overbrace{\left[\mathbf{t}^y_{center} \cdot (\mathbf{r}_{center} - \mathbf{r}_{neigh})\right] \mathbf{t}^y_{center}}^{\text{y-component}}$$

9 short texts, 63 pages of lecture notes from Professor Marty Sereno's lecture series Neuroimaging (University of California 2006), explaining how MRI images are obtained.

Conny Blom, A Grey Matter 2013/2021

Project for the 5th Moscow Biennale, *Special Project There is a Visitor # 13*

CAC 018
2021



A Grey Matter